Memetic Simulated Annealing for the GPS Surveying Problem

Stefka Fidanova, Enrique Alba, and Guillermo Molina

IPP – Bulgarian Academy of Sciences, Acad. G. Bonchev str. bl.25A,
1113 Sofia, Bulgaria
stefka@parallel.bas.bg
Universidad de Málaga, E.T.S.I. Informática, Grupo GISUM (NEO)
Málaga, España
{eat,guillermo}@lcc.uma.es

Abstract. In designing Global Positioning System (GPS) surveying network, a given set of earth points must be observed consecutively (schedule). The cost of the schedule is the sum of the time needed to go from one point to another. The problem is to search for the best order in which this observation is executed. Minimizing the cost of this schedule is the goal of this work. Solving the problem for large networks to optimality requires impractical computational times. In this paper, several Simulated Annealing (SA) algorithms are developed to provide near-optimal solutions for large networks with bounded computational effort.

1 Introduction

A Global Positioning System (GPS) is a satellite-based radio-navigation system that permits land, sea, and airborne users to determine their three dimensional position, velocity, and time. This service is always available at any time and under any weather condition. In addition, satellite navigation systems have an impact in geoscience, in particular on surveying work in quick and effective determining positions and changes in positions networks. Measuring requires that the survey crew physically passes through all the intervening terrain to measure the distance between any two adjacent points. The most widely known space systems are: the American NAVSTAR global positioning system, the Russian GLObal Navigation Satellite System (GLONASS), and the forthcoming European satellite navigation system (GALILEO).

In this paper, we investigate the use of GPS to establish surveying networks. GPS satellites continuously transmit radio signals to the Earth while orbiting it. A receiver, with unknown position on Earth, has to detect and convert the signals received from all of the satellites into useful measurements. These measurements would allow a user to compute a three-dimensional coordinate position: the location of the receiver.

Solving this large problem to optimality requires a very high computational time. Therefore, new methods are needed to provide near-optimal solutions for

S. Margenov, L.G. Vulkov, and J. Waśniewski (Eds.): NAA 2008, LNCS 5434, pp. 281–288, 2009.

large networks within an acceptable amount of computational effort. These techniques are usually based on structured metaheuristics [3,5,10]. In this paper, several Simulated Annealing (SA) algorithms are introduced and applied on test problems with different dimension.

The organization of the paper is as follows. The general framework for a GPS surveying network problem as a combinatorial optimization problem is described in Section 2. Then, different search strategies for SA are explained in Section 3. The presentation of the initial temperature as a function of the initial solution cost is investigated in Section 4. The numerical results are presented and discussed in Section 5. The paper ends with conclusions and directions for future research.

2 Problem Description

The GPS network can be defined as set of stations $(a_1, a_2, \dots a_n)$, which are co-ordinated by placing receivers $(X1, X2, \dots)$ on them to determine sessions $(a_1a_2, a_1a_3, a_2a_3, \dots)$ between them. The GPS surveying problem consists in searching for the best order in which these sessions can be organized to give the best schedule. Thus, the schedule can be defined as a sequence of sessions to be observed consecutively. The solution is represented by a linear graph with weighted edges. The nodes represent the stations and the edges represent the moving cost. The objective function of the problem is the cost of the solution, which is the sum of the costs (time) to move from one point to another one, $C(V) = \sum C(a_i, a_j)$, where a_ia_j is a session in solution $V = (a_1, a_2, \dots, a_n)$.

The initial data is a cost matrix, which represents the cost of moving a receiver from one point to another. The cost could be evaluated purely upon the time or purely upon the distance; for more details see Dare [3]. This problem resembles the Traveling Salesman Problem (TSP). The main difference is that the TSP requires a closed path through all the nodes, where the initial and final nodes are the same, whereas the GPS problem finds an open path. Thus the strategies to solve GPS surveying problem can be different from these for TSP.

3 Simulated Annealing for the GPS Surveying Problem

SA is a heuristic method that has been implemented to obtain good solutions of an objective function defined on a number of discrete optimization problems [7]. This method has proved to be a flexible local search method and can be successfully applied to the majority of real-life problems [1,4,9]. The fundamental idea is to allow moves resulting in solutions of worse quality than the current solution in order to escape from local minima.

Simulated Annealing is a stochastic heuristic method which explores the solution space using a stochastic hill-climbing process. SA is inspired by the Metropolis scheme [8]. An initial state of a thermodynamic system was chosen with energy E and temperature T, holding T constant the initial configuration is perturbed and the change of energy dE is computed. The current state of the thermodynamic system is analogous to the current solution to the combinatorial problem,

```
Simulated Annealing

t:=0;
Initialize(V) - initial solution;
Initialize(T) - temperature;
Initialize(F) - cooling rate;
while not end_condition(t,V) do
        while not cooling_condition(t)
                V' := Choose_neighbor(V);
                Δ = C(V')-C(V);
                if Δ < 0 then
                   V := V';
                end if
                else
                   Generate_random number Θ
                   if e^(-Δ/T) > Θ
                           V:=V';
                   end if
                end else
                t := t+1;
        end while
        Cooldown(T);
end while
```

Fig. 1. Pseudocode for SA

the energy equation for the thermodynamic system is analogous to the objective function, and the ground state is analogous to the global minima. The algorithm starts by generating an initial solution and by initializing the so-called temperature parameter T. The temperature is decreased during the search process, thus at the beginning of the search the probability of accepting uphill moves is high and it gradually decreases. The structure of the SA algorithm is shown in Figure 1. The key objective of this paper is to find an effective solution in a short period of time with close to lowest cost for a given GPS network using Simulated Annealing.

The basic parts of SA are the initial solution construction and current solution perturbation. Two kinds of initial solutions, greedy and random, are used in this paper. To construct the greedy initial solution we start from a random node. The next node is the closest (cheapest) node to the current, which has not yet been included in the solution.

SA is a Local Search (LS) based method. The main concept of LS is searching the local neighborhood of the current solution [11]. In general, neighborhood for large-size problems can be much complicated to search. Therefore, LS attempts to improve a current schedule V to a GPS network by a small series of local improvements. A move generation is a transition from a schedule V to another one $V' \in I(V)$ in one step (iteration). The returned schedule V' may not be optimal, but it is the best schedule in its local neighborhood $I(V)$. A local optimal schedule is a schedule with the local minimal cost value.

In this paper several local search procedures $L(k, l)$ are applied, where k is the number of generated neighbor solutions and l is the number of used perturbation method. This raises a concept that can be called *memetic SA* because of the potential benefits of focusing on an internal local search step instead of the (usually) light perturbation of the canonical SA. Also, including problem knowledge in the initial generation of the solution is in the philosophy of memetic approaches [6]. As a consequence, for regular SA $k = 1$. The prepared by us perturbations are as follows:

1. Nodes Sequential Swaps: **for** $i = 1$ **to** $n - 1$; **for** $j = i + 1$ **to** n; swap a_i and a_j; [10];
2. Nodes Random Swaps: two nodes are chosen randomly and are swapped;
3. Randomly Delete an Edge: let the current solution is $(a_1 , a_2 , \ldots , a_i , a_{i+1} , \ldots , a_n)$. The edge $(i, i + 1)$ is randomly chosen and deleted. The new solution is $(a_{i+1} , \ldots , a_n , a_1 , \ldots , a_i)$;
4. Greedy Delete an Edge: The longest (most expensive) edge is deleted. The new solution is constructed as in upper case;
5. Randomly Delete 2 Edges: Let the current solution is $(a_1 , a_2 , \ldots , a_i , a_{i+1} , \ldots , a_j , a_{j+1} , \ldots , a_n)$. The edges $(i, i + 1)$ and $(j, j + 1)$ are randomly chosen and deleted. The new solutions are $(a_{i+1} , \ldots , a_j , a_1 , \ldots , a_i , a_{j+1} , \ldots ,a_n)$, $(a_{j+1} , \ldots , a_n , a_{i+1} , \ldots , a_j , a_1 , \ldots , a_i)$, $(a_1 , \ldots , a_i , a_{j+1} , \ldots , a_n , a_{i+1} , \ldots , a_j)$;
6. Greedy Delete 2 Edges: The two longest edges are deleted. The new solutions are prepared as in the upper case.

When the cost of the best neighbor solution is equal to the cost of the current solution, we choose randomly the new current solution from the set of neighbors. The aim is to prevent $\Delta = 0$ and thus to prevent repetition of the same solutions. It is a way for diversification of the search process.

4 Parameter Settings

The parameter settings play a crucial role in the behavior of SA. The main SA parameters are initial temperature T_0 and cooling rate F. Most authors use a constant value for T_0, which is not related to the solved problem [9,10]. In [2] the temperature starts at T_0 close to ∞ and decreases very quickly. The goal of the mentioned work is to quicken the search process.

Our idea is to set the initial temperature T_0 to be a linear function of the cost of the initial solution, $T_0 = K * C(V_0)$, where K is a parameter. We decided T_0 to be a linear function of $C(V_0)$, because Δ is a linear function of $C(V)$. The acceptance probability depends on the difference Δ between the costs of the current and candidate solutions. Thus, if Δ is large, the probability of accepting the candidate solution is higher. The main problem is what large means. For example: Let $C(V) = 100$ and $\Delta = 20$ thus Δ is 20% of the value of $C(V)$ and Δ is large with respect to $C(V)$. Let $C(V) = 100\,000$ and $\Delta = 20$, thus Δ is 0.02% of the value of $C(V)$ and Δ is not large with respect to $C(V)$. For most problems the expected cost of the optimal solution is unknown, so our proposal is to automatically set the initial temperature to be proportional to the initial solution cost.

$$\frac{\Delta}{T} = \frac{C(V') - C(V)}{K * C(V_0)} \tag{1}$$

The other important parameter is the cooling parameter F. In previous works [2,7] it is recommended to use a value for the cooling parameter of 0.85 or higher in order to guarantee the theoretical convergence of SA to the global optima. In the next section the values of parameters K and F are investigated to understand their influence on the SA algorithm applied on the GSP surveying problem.

5 Experimental Results

In this section we analyze the experimental results obtained using the various SA algorithms described in previous sections. As a test problems we use real data from Malta and Seychelles GPS networks. The Malta GPS network is composed of 38 sessions and the Seychelles GPS network is composed of 71 sessions. We use 6 larger test problems too, from http://www.informatik.uni-heidelberg.de/ groups/ comopt/ software/ TSLIB95/ ATSP.html. These test problems range from 100 to 443 sessions.

For every experiment, the results are obtained by performing 30 independent runs, then averaging the fitness values obtained in order to ensure statistical confidence of the observed difference. Analysis of the data using ANOVA/Kruskal-Wallis test plus Multicompare function has been used to get statistical confidence of the results with a confidence level of 95% .

First the SA algorithms with random initial solution and various kinds of current solution perturbation are applied to all test problems with the same parameters as follows: initial temperature is 198, temperature decay is 0.85, Markov's chain is 1200, number of evaluations is 116000.

Comparing all the perturbation methods (Table 1), the best results (cost of the schedule, in bold) are obtained by $L(1,5)$, except for the Seychelles problem. With $L(1,5)$ perturbation the diversification is larger than with the rest of the techniques. We decided to further analyze the effects of using a greedy initial solution with only $L(1,5)$, since it achieves the best results.

Table 1. Comparison of simulated annealing algorithms with random initial solution, applied to various types of GPS networks

Test	sessions	LS(1,2)	LS(1,3)	LS(1,4)	LS(1,5)	LS(1,6)
Malta	38	1405	1285	1285	**1021**	1285
Seychelles	71	994	994	994	1052	**949**
rro124	100	206653	205643	205643	**48604**	200724
ftv170	170	7101	6908	6908	**6043**	7001
rbg323	323	6412	6396	6396	**5599**	6363
rbg358	358	7068	7050	7050	**6205**	7035
rbg403	403	7945	7923	7923	**6996**	7858
rbg443	443	8694	8486	8486	**7592**	8685

Table 2. Influence of the initial solution on the achieved results using L(1,5)

Tests stations	Malta 38	Seychelles 71	rro124 100	ftv170 170	rbg323 323	rbg358 358	rbg403 403	rbg443 443
random	1021	1052	48604	6043	5599	6205	6996	7592
greedy	**960**	**1037**	**43836**	**3980**	**1731**	**1820**	**3533**	**3891**

Comparing the influence of the initial solution generation (Table 2), we can conclude that starting from a greedy initial solution achieves lower cost results than starting from a random initial solution. The greedy initial solution is much better for most of the problems and gives the possibility for the algorithm to start from a solution which is closer to the optimal one, therefore our memetic algorithm approach represents a step forward in the techniques for solving this problem.

Next, we include in this study various kinds of local search procedures to improve the algorithm performance. To keep the running time low, the neighbor set consists of as many solutions as the number (n) of sessions. The number of iterations is equal to $10 \times n$.

Table 3. Simulated annealing algorithms plus local search applied to various types of GPS networks, n is equal to the number of sessions

Test	sessions	L(n,1)	L(n,2)	L(n,3)	L(n,4)	L(n,5)	L(n,6)
Malta	38	1345	**903**	1285	1285	**903**	1285
Seychelles	71	986	**930**	994	994	937	949
rro124	100	125606	56023	205463	205463	**38230**	191931
ftv170	170	6942	4994	6908	6908	**4599**	6605
rbg323	323	3855	1898	6396	6396	**1429**	5808
rbg358	358	3354	1858	7050	7050	**1335**	4996
rbg403	403	4845	2929	7923	7923	**2528**	5747
rbg443	443	5715	3242	8486	8486	**2844**	6065

Comparing SA with different local search procedures (Table 3) the best results are obtained by $L(n, 5)$, except for the Malta and Seychelles test problems. For these test instances there are no significant differences between the results obtained by $L(n, 2)$ and $L(n, 5)$, but the standard deviations of $L(n, 5)$ are three times smaller than using $L(n, 2)$, which suggests its higher robustness.

We test the algorithm for all pairs (K, F),where K is from the set {0.002, 0.01, 0.1, 0.25, 0.50, 0.75, 1} and F is from the set {0.85, 0.90, 0.95, 0.99}, for

Table 4. Influence of the initial temperature parameter K

Test	sessions	K=0.002	K=0.01	K=0.1	K=0.25	K=0.50	K=0.75	K=1
Malta	38	**939**	974	1030	1035	1035	1035	1035
Seychelles	71	**965**	1021	1036	1038	1040	1040	1040
rro124	100	**46622**	47106	47879	47879	47879	47879	47879
ftv170	170	**3706**	3926	3980	3980	3980	3980	3980
rbg323	323	**1726**	1731	1738	1738	1738	1738	1738
rbg358	358	**1808**	1818	1820	1820	1820	1820	1820
rbg403	403	**3530**	3532	3533	3533	3533	3533	3533
rbg443	443	**3888**	3891	3891	3892	3892	3892	3892

all test problems. We observed that the achieved costs are statistically similar went we fixed the value of K and changed the value of F.

In Table 4 we show the achieved costs for every test problem for different values of the temperature parameter K and $F = 0.85$. Analyzing the results, we conclude that, the algorithm in which $K = 0.002$ outperforms the others. Minimal costs are obtained by $K = 0.002$ for most of the test problems except tests rbg323, rbg403 and rbg443. For these three test problems there is no significant difference between the costs obtained by $K = 0.002$ and $K = 0.01$. For $K \geq 0.1$ the achieved costs are statistically similar. When the parameter $K \geq 0.1$, the initial temperature is too large and the probability to accept an inferior solution is too high. In this case the algorithm operates like random search. When the parameter $K < 0.002$, the value of the initial temperature is too small and it is difficult for the algorithm to climb the hills. In this case the algorithm proceeds in a greedy way. Therefore, we decided not to decreases the value of K.

6 Conclusion

The GPS surveying problem is addressed in this paper. Instances containing from 38 to 443 sessions have been solved using Simulated Annealing algorithms with various kinds of current solution perturbation and two kinds of initial solution construction. For solution improvement several local search procedures are developed and applied. The solution perturbation that deletes two randomly chosen edges and includes two new outperforms others. The results obtained using a greedy initial solution outperform those obtained using a random one. The combination of the SA with local search procedures has shown improvements of the results. The best results are achieved by using LS deleting two edges randomly and including two new ones, $L(n, 5)$ where n is equal to the number of sessions. The initial temperature is represented as a linear function of the initial solution cost and the best results are achieved when $T_0 = 0.002 * C(V_0)$. The cooling parameter F has vary low rate of influence.

In future work, other metaheuristics algorithms will be developed, applied and compared.

Acknowledgments. This work has been partially funded by the Spanish Ministry of Science and Technology and FEDER under contract TIN2005-08818-C04-01 (the OPLINK project). We also acknowledge partial funding from the Spanish Ministry of Industry and European EUREKA-CELTIC under contract FIT-330225-2007-1, label CP3-005 (the CARLINK project). Guillermo Molina is supported by grant AP2005-0014 from the Spanish government.

References

1. Arts, E., Van Loarhoven, P.: Statistical Cooling: A General Approach to Combinatorial Optimization Problems. Phillips Journal of Research 40, 193–226 (1985)
2. Chen, T.-C., Chang, Y.-W.: Modern Floorplaning Based on B* Fast Simulated Annealing. IEEE Trans. on Computer Aided Design 25(4), 637–650 (2006)
3. Dare, P.J., Saleh, H.A.: GPS Network Design:Logistics Solution Using Optimal and Near-Optimal Methods. J. of Geodesy 74, 467–478 (2000)
4. Dowsland, K., Thomson, J.: Variants of Simulated Annealing for the Examination Timetabling Problem. Annals of OR 63, 105–128 (1996)
5. Fidanova, S.: An Heuristic Method for GPS Surveying Problem. In: Shi, Y., van Albada, G.D., Dongarra, J., Sloot, P.M.A. (eds.) ICCS 2007. LNCS, vol. 4490, pp. 1084–1090. Springer, Heidelberg (2007)
6. Hart, W.E., Krasnogor, N., Smith, J.E. (eds.): Recent Advances in Memetic Algorithms. Studies in Fuzziness and Soft Computing, vol. 166 (2005)
7. Kirkpatrick, S., Gellat, C.D., Vecchi, P.M.: Optimization by Simulated Annealing. Science 220, 671–680 (1983)
8. Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., Teller, E.: Equation of State Calculations by Fast Computing Machines. J. of Chem Phys. 21(6), 1087–1092 (1953)
9. Rene, V.V.: Applied Simulated Annealing. Springer, Berlin (1993)
10. Saleh, H.A., Dare, P.: Effective Heuristics for the GPS Survey Network of Malta: Simulated Annealing and Tabu Search Techniques. Journal of Heuristics 7(6), 533–549 (2001)
11. Schaffer, A.A., Yannakakis, M.: Simple Local Search Problems that are Hard to Solve. Society for Industrial Applied Mathematics Journal on Computing 20, 56–87 (1991)