Numerical Analysis Of Multilevel Milstein Scheme Without Lévy Areas

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The multilevel Monte Carlo method can give a great improvement in computational efficiency compared to the standard Monte Carlo method when estimating the expected value of some functional of the solution to an SDE. This is achieved through the use of multiple levels of resolutions, expressing the expectation on the finest level as equal to the expectation on the coarsest level of approximation plus a telescoping sum of expected corrections between neighbouring levels of approximation. Because of strong convergence, the corrections have a low variance, and so relatively few samples are required on the finer levels to accurately estimate the expected correction.

This has motivated the use of the first order Milstein discretisation in preference to the simpler Euler-Maruyama discretisation. Previous work has demonstrated and analysed its superiority for scalar SDEs with a variety of path-dependent functionals relevant to mathematical finance.

However, when applying the approach to multi-dimensional SDEs there is the problem of the Lévy areas which are in general required for the Milstein discretisation. Previous work in this regard has demonstrated numerically that by using an antithetic treatment one can neglect the Lévy area terms and still construct a multilevel estimator which has excellent variance properties, despite the strong convergence being no better than for the Euler-Maruyama discretisation.

In this paper, we will present for the first time the numerical analysis of this scheme, deriving the rate at which the multilevel correction variance converges to zero.