

Strong Convergence and Stability of Numerical Methods for Non-Linear SDEs with Applications to MLMC Simulation of Financial Models.

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In many financial engineering applications, one is interested in the expected value of a financial derivative whose payoff depends upon the solution of a stochastic differential equation. Using a simple Monte Carlo method with a numerical discretization with first order weak convergence, to achieve a root-mean-square error of $O(\epsilon)$ would require $O(\epsilon^{-2})$ independent paths, each with $O(\epsilon^{-1})$ time steps, giving a computational complexity which is $O(\epsilon^{-3})$. Recently, Giles [Giles, 2008] introduced a Multilevel Monte Carlo (MLMC) estimator, which enables a reduction of this computational cost to $O(\epsilon^{-2})$. In order to achieve this superior property of the MLMC estimator, the numerical discretization of a SDEs under consideration requires certain convergence properties, namely that the numerical approximation strongly converge to the solution of the SDEs with order 1. This carries some difficulties. First of all, it is well known that it impossible to obtain an order of convergence higher than 0.5 without a good approximation of the Levy areas (which are very expensive to simulate) [Cameron, 1980]. Second, convergence and stability of numerical methods are well understood for SDEs with Lipschitz continuous coefficients, whereas most financial SDEs violates these conditions. It was demonstrated, that for SDEs with non-Lipschitz coefficients using the classical methods, we may fail to obtain numerically computed paths that are accurate for small step-sizes, or to obtain qualitative information about the behaviour of numerical methods over long time intervals [Jentzen, 2011]. Our work addresses both of these issues, giving a customized analysis of the most widely used numerical methods. In this work we generalize the current theory of strong convergence rates for Euler-Maruyama-type schemes for some highly non-linear multidimensional SDEs which appear in finance. We also generalize Fundamental Theorem by Milstein on Strong Convergence of the general numerical approximation in this

non-linear setting. As an application we will show that a new MLMC estimator that enables us to avoid simulation of Levy areas without affecting the required computational cost of order $O(\epsilon^{-2})$ can be applied to the rich family of SDEs. We support our theoretical results with the simulations of expected values of options with various payoffs.