The Efficiency Study of Splitting and Branching in Monte Carlo Method

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The Monte Carlo solution of second-kind integral equations is usually based on the von Neumann–Ulam scheme, since these equations are related to homogeneous Markov chains terminating with probability one. In this talk we study the computational complexity of the use of standard weight collision estimator and some of its modifications with splitting and branching for estimating the solution of the second-kind integral equation. In general the use of trajectory splitting diminishes the variance of estimator and increases the computation time. In this talk we present the optimal value for splitting of the trajectory at some time in Markov chain. However it turned out that the use of trajectory splitting once at the first time in Markov chain provides the lower computational efficiency as compared with the algorithm without splitting.

The basic characteristic of weight collision estimator is a random weight that is multiplied by the ratio of the equations corresponding kernel to the transition density after every transition in the simulated Markov chain. If the weight values do not exceed 1, then the mean square (and, hence, the variance) of the standard collision estimate for the functional is finite. Otherwise we can branch the simulated trajectory to random number of branches with expectation that is equal to the weight. We show that the use of such branching provides the lower computational efficiency as compared with the algorithm without branching.