Bias evaluation and reduction for sample-path optimization

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We consider general nonlinear stochastic problems of the form

$$\min_x f(E[g(x, \boldsymbol{\xi}]),$$

where $\boldsymbol{\xi}$ is some random variable. $\boldsymbol{\xi}$ can be continuous or discrete, but then with a large number of possible realizations. This problem is more general that the traditional setting, where f would be the identity function. Moreover, f and g are here possibly nonconvex, and we assume that no close expression is available for $E[g(x, \boldsymbol{\xi}]$. A traditional approach is approximate this expectation by Monte Carlo, leading to the approximate program

$$\min_x f\left(\frac{1}{N}\sum_{i=1}^{N} g(x, \xi_i)\right),$$

where $\{\xi_1, \xi_2, \ldots, \xi_N\}$ is an i.i.d. sample over $\boldsymbol{\xi}$. Due to nonlinearities, this approximation is usually biased for finite N. We review the conditions under which this approximation is consistent when N grows to infinity, and consider various strategies to reduce the introduced bias for finite N. We mainly focus on techniques relying on Taylor expansion (under appropriate smoothness conditions), and compare them to other approaches, as bootstrap bias estimation.

Any bias correction can however increase the variance of the approximation. We therefore consider information as mean-square error in order to evaluate its impact, and examine how we can reduce it by introducing negative correlation between the sample average approximation and the bias correction. In order to achieve this goal, we take inspiration from simulation strategies, for instance common random numbers. Various numerical examples will be exhibited in order to evaluate the strategy, with a special emphasis on advanced discrete choice models estimation. We also briefly empirically examine the influence of the bias originating from the optimization process, and give some novel insight on the interactions between these two kinds of bias.