Collision Densities And Mean Residence Times For d-dimensional Exponential Flights

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In this work we analyze some aspects of *exponential flights*, a stochastic process that governs the evolution of many random transport phenomena, such as neutron propagation, chemical/biological species migration, or electron motion. We introduce a general framework for d-dimensional setups, and emphasize that exponential flights represent a deceivingly simple system, where in most cases closed-form formulas can hardly be obtained. We derive a number of novel exact (where possible) or asymptotic results, among which the stationary probability density for $2d$ systems, a long-standing issue in Physics, and the mean residence time in a given volume. Bounded or unbounded, as well as scattering or absorbing domains are examined, and Monte Carlo simulations are performed so as to support our findings.

Random walks are widely used in Physics so as to model the features of transport processes where the migrating (possibly massless) particle undergoes a series of random displacements as the effect of repeated collisions with the surrounding environment. While much attention has been given to random walks on regular Euclidean lattices, and to the corresponding scaling limits, less has been comparatively devoted to the case where the direction of propagation can change continuously at each collision. Such processes, which are intimately connected to the Boltzmann equation, have been named *random flights*, and play a prominent role in the description of, among others, neutron or photon propagation through matter, chemical and biological species migration, or electron motion in semiconductors.

Within the simplest formulation of this model, which was originally proposed by Pearson (1905) and later extended by Kluyver (1906) and Rayleigh (1919), it is assumed that particles perform random displacements ('flights') along straight lines, and that at the end of each flight (a 'collision' with the surrounding medium) the direction of propagation changes at random.

When the number of transported particles is much smaller than the number of the particles of the interacting medium, so that inter-particles collisions can be safely neglected, it is reasonable to assume that the probability of interacting with the medium is Poissonian. For the case of neutrons in a nuclear reactor, e.g., the ratio between the number of transported particles and the number of interacting nuclei in a typical fuel/moderator configuration is of the order of 10^{-11}, even for high flux reactors. It follows that flight lengths between subsequent collisions are exponentially distributed (hence we will call this process *exponential flights* in the following. We assume that collisions can be either of scattering or absorption type. At each scattering collision, the flight direction changes at random, whereas at absorption events the particle disappears and the flight terminates. Each flight can be seen as a random walk in the phase space of position $\mathbf{r}$ and direction ω in a d-dimensional setup.

Along with the development of Monte Carlo methods, numerical solutions for the particle density to complex three-dimensional linear/nonlinear transport problems coming from applied sciences are becoming accessible to an high degree of accuracy: criticality calculations in reactor cores, scattering and absorption in heated plasmas, propagation through anisotropic scattering centers in atmosphere or fluids, and charge transport in semiconductors under external fields, only to name a few. Nonetheless, even for the simplest systems, many theoretical questions remain without an answer, so that the study of exponential flights has attracted a renovated interest in recent years. In particular, it has been emphasized that the dimension d deeply affects the nature of the particle density, and prevents in most cases from obtaining explicit results. The aim of our work is to investigate exponential flights in a generic d-dimensional setup, under simplifying hypotheses. Here, we will mostly focus on establishing insightful relationships between space, time and the statistics of particle collisions within a given volume. A number of new results will be derived, concerning unbounded, bounded, scattering as well as absorbing domains.