Pointwise Approximation of Stochastic Heat Equations with Additive Noise

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We consider stochastic heat equations on the spatial domain $(0,1)^d$ with additive nuclear noise as well as additive space-time white noise, and we study approximation of the mild solution at a fixed time point. The error of an algorithm is defined by the average L_2-distance between the solution and its approximation. The cost of an algorithm is defined by the total number of evaluations of one-dimensional components of the driving Brownian motion at arbitrary time nodes. We want to construct algorithms with an (asymptotically) optimal relation between cost and error. Furthermore we wish to determine the asymptotic behaviour of the corresponding minimal errors.

We show how the minimal errors depend on the spatial dimension d and, in the case of nuclear noise, on the decay of the eigenvalues of the associated covariance operator.

Asymptotic optimality is achieved by a spectral Galerkin approximation together with a nonuniform time discretization. This optimality cannot be achieved by uniform time discretizations, which are frequently studied in the literature.

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