Sensitivity Analysis of Compact Models in Nanodevice Modeling

Rayna Georgieva[1]
(in collaboration with Asen Asenov[2] Binjie Cheng[2]
Ivan Dimov[1] Urban Kovac[2] Campbell Millar[2])

[1] Institute of Information and Communication Technologies - Bulgarian Academy of Sciences

[2] The University of Glasgow, Rankine Building, Glasgow, United Kingdom

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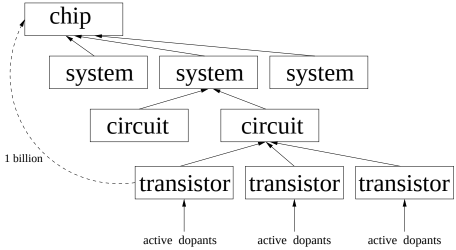
- Introduction
  - Motivation
  - Statistical variability in transistors
  - Sensitivity analysis technique
- Numerical experiments
- Open problems

- Motivation
  - Simulation models are used (in diagnostic or prognostic fashion) in many fields to understand complex phenomena (natural or social) and consequently as tools to support decisions and policy.
  - Our knowledge is often flawed by uncertainties (partly irreducible, largely unquantifiable), imperfect understanding, subjective values.
  - We need tools to scrutinize uncertainties in model inputs, assumptions, models structures, to see how they propagate and affect inferences (that are used for policy decisions).



- Compact model parameters (in circuit simulations)
  - Compact model parameter generation
  - Statistical compact model parameter extraction

- Definition of sensitivity analysis (SA)

  "*The study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input.*"

  A. Saltelli et al. *Global Sensitivity. The Primer*. John Wiley & Sons, Ltd (2008).

- Metamodeling
- Sensitivity analysis techniques
  - Local approach (one-at-a-time experiments)
  - Screening methods
  - Variance-based methods - Sobol' approach, FAST
  - Derivative-based global sensitivity measures
- Monte Carlo / quasi-Monte Carlo methods
  - Plain Monte Carlo Algorithm
  - Adaptive Monte Carlo Algorithm
  - Sobol' Quasi Monte Carlo Algorithm
  - Monte Carlo Algorithm Based on Sobol' Sequences
- Pseudo / quasi-random number generators

- The mathematical model

$$\mathbf{u} = f(\mathbf{x}), \quad \text{where} \quad \mathbf{x} = (x_1, x_2, \ldots, x_d) \in U^d \equiv [0,1]^d$$

is a vector of inputs with a joint p.d.f. $p(\mathbf{x}) = p(x_1, \ldots, x_d)$.

- The mathematical model

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is a vector of inputs with a joint p.d.f. $p(\mathbf{x}) = p(x_1, \ldots, x_d)$.

- Total Sensitivity Index of input parameter $x_i, \quad i \in \{1, \ldots, d\}$:

$$S_{x_i}^{tot} = S_i + \sum_{l_1 \neq i} S_{il_1} + \sum_{l_1, l_2 \neq i, l_1 < l_2} S_{il_1 l_2} + \ldots + S_{il_1 \ldots l_{d-1}},$$

where

S_i - *the main effect (first-order sensitivity index)* of x_i and
$S_{il_1 \ldots l_{j-1}}$ – j^{th} *order sensitivity index for parameter* x_i $(2 \leq j \leq d)$.

ANalysis Of VAriances (ANOVA) HDMR of a square integrable function $f(\mathbf{x})$:

$$f(\mathbf{x}) = f_0 + \sum_{\nu=1}^{d} \sum_{l_1<\ldots<l_\nu} f_{l_1\ldots l_\nu}(x_{l_1}, x_{l_2}, \ldots, x_{l_\nu}), \text{ where } f_0 = const,$$

and $$\int_0^1 f_{l_1\ldots l_\nu}(x_{l_1}, x_{l_2}, \ldots, x_{l_\nu}) \mathrm{d}x_{l_k} = 0, \quad 1 \le k \le \nu, \ \ \nu = 1, \ldots, d.$$

The functions in the right-hand side are defined in a unique way:

- $f_0 = \int_{U^d} f(\mathbf{x}) \mathrm{d}\mathbf{x}, \quad f_{l_1}(x_{l_1}) = \int_{U^{d-1}} f(\mathbf{x}) \prod_{k\neq l_1} \mathrm{d}\mathbf{x}_k - f_0, \ l_1 \in \{1, \ldots, d\}$
- $\int_{U^d} f_{i_1\ldots i_\mu} f_{j_1\ldots j_\nu} \, \mathrm{d}\mathbf{x} = 0, \ (i_1, \ldots, i_\mu) \neq (j_1, \ldots, j_\nu), \ \mu, \nu \in \{1, \ldots, d\}.$

Definition (Sobol')

$$S_{l_1 \ldots l_\nu} = \frac{\mathbf{D}_{l_1 \ldots l_\nu}}{\mathbf{D}}, \quad \nu \in \{1, \ldots, d\},$$

where

- partial variances $\mathbf{D}_{l_1 \ldots l_\nu} = \int f^2_{l_1 \ldots l_\nu} \mathrm{d}x_{l_1} \ldots \mathrm{d}x_{l_\nu}$,
- total variance $\mathbf{D} = \int_{U^d} f^2(\mathbf{x}) \mathrm{d}\mathbf{x} - f_0^2$, $\quad \mathbf{D} = \sum_{\nu=1}^{d} \sum_{l_1 < \ldots < l_\nu} \mathbf{D}_{l_1 \ldots l_\nu}$,

and the following properties hold:

- $S_{l_1 \ldots l_s} \geq 0$, $\quad \sum_{s=1}^{d} \sum_{l_1 < \ldots < l_s}^{d} S_{l_1 \ldots l_s} = 1$.

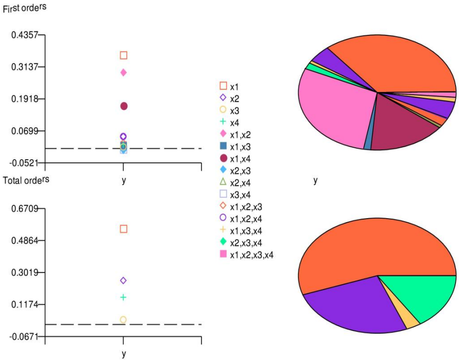
- BSIM4 compact model
- Statistical variability
- Sensitivity analysis of a submodel
  - V_{th0} - basic long-channel threshold voltage parameter
  - U_0 - low-field mobility parameter
  - R_{dsw} - basic source/drain resistance parameter
  - D_{sub} - drain-induced barrier-lowering (DIBL) parameter

- Random number generation - SIMLAB v2.2
- Produce a database with model outputs
- Computing Sobol' sensitivity indices - SIMLAB v2.2

SIMLAB: http://simlab.jrc.ec.europa.eu/

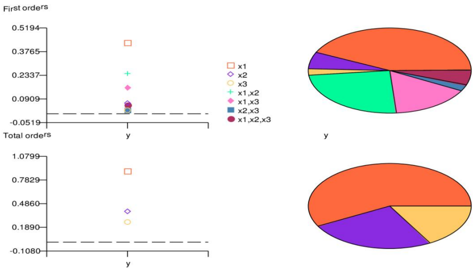
Sobol indices

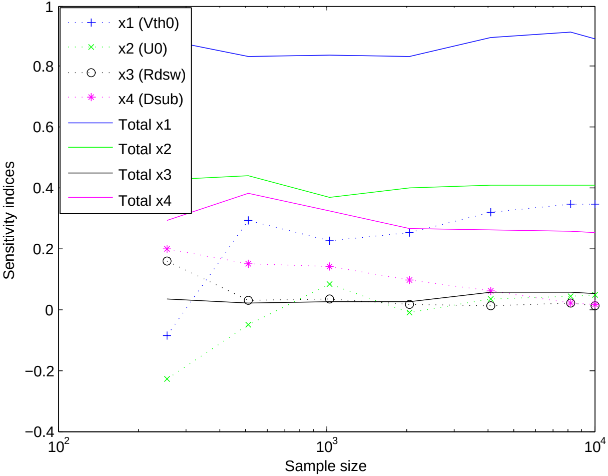
	y	y-Closed
x1	0.359418	0.359418
x2	0.047362	0.047362
x3	0.008886	0.008886
x4	0.017300	0.017300
x1,x2	0.290449	0.697229
x1,x3	0.014991	0.383295
x1,x4	0.165470	0.542188
x2,x3	-0.00857	0.047677
x2,x4	0.011997	0.076659
x3,x4	-0.00507	0.021117
x1,x2,x3	0.022358	0.734893
x1,x2,x4	0.047931	0.939927
x1,x3,x4	0.011986	0.572982
x2,x3,x4	8.92e-005	0.071994
x1,x2,x3,x4	0.015403	1.000000
Total x1	0.900432	
Total x2	0.418538	
Total x3	0.049705	
Total x4	0.255231	

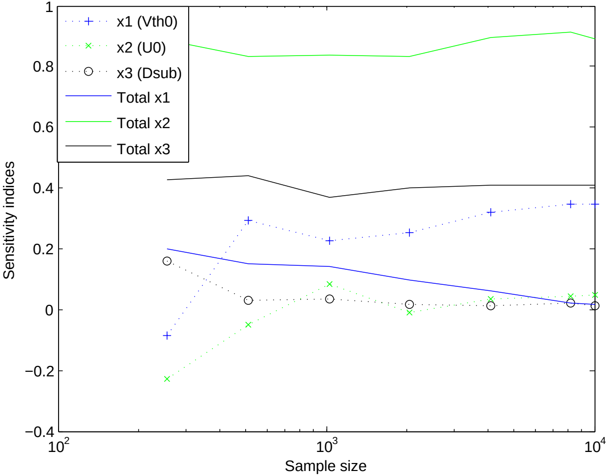


Sobol indices

	y	y-Closed
x1	0.429220	0.429220
x2	0.066087	0.066087
x3	0.022617	0.022617
x1,x2	0.245330	0.740638
x1,x3	0.157343	0.609180
x2,x3	0.023386	0.112090
x1,x2,x3	0.056016	1.000000
Total x1	0.892491	
Total x2	0.386237	
Total x3	0.258302	




x1 (Vth0)
x2 (U0)
x3 (Rdsw)
x4 (Dsub)
Total x1
Total x2
Total x3
Total x4
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
Sensitivity indices
10^2
10^3
10^4
Sample size


1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
Sensitivity indices
10^2
10^3
10^4
Sample size
x1 (Vth0)
x2 (U0)
x3 (Dsub)
Total x1
Total x2
Total x3

- Uniform or normal distribution of inputs?
- Approximation of model database
- Orthogonality (independence) of inputs