

# Group Object Tracking with Sequential Monte Carlo Methods Based on a Parameterised Likelihood Function

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- SMC Framework
- Likelihood Function Based on Surface Parametrisation
- Parameterisation of the Visible Surface

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## Motivation



## Background (1)

-  W. Koch and M. Feldmann. Cluster tracking under kinematical constraints using random matrices. *Robotics and Autonomous Systems*, 57(3):296 – 309, 2009.
-  M. Baum, M. Feldmann, D. Fränken, U. D. Hanebeck, and W. Koch. Extended Object and Group Tracking: A Comparison of Random Matrices and Random Hypersurface Models. *In LNCS, 2010*.
-  K. Gilholm and D. Salmond. Spatial Distribution Model for Tracking Extended Objects. *IEE Proc.-Radar, Sonar Navig.*, 152(5):364–371, 2005.



## Background (2)

-  M. Baum and U. D. Hanebeck. Extended Object Tracking based on Combined Set-Theoretic and Stochastic Fusion. *In Proc. of the International Conf. on Information Fusion, 2009.*
-  D. Angelova and L. Mihaylova. Extended Object Tracking Using Monte Carlo Methods. *IEEE Transactions on Signal Processing, 56(2):825–832, 2008.*
-  A. Gning, L. Mihaylova, S. Maskell, S.K. Pang, and S. Godsill. Group Object Structure and State Estimation With Evolving Networks and Monte Carlo Methods. *IEEE Trans. on Signal Processing, 59(4):1383 –1396, 2011.*
-  PHD Filters - [Mahler, Vo, Ristic, Willett, Clark, Koch, Gustafsson, etc.](#)

## Goals and Contributions

### Goals

Set a framework for group object tracking based on:

- nonlinear system dynamics model,
- nonlinear measurement,
- arbitrary noise distribution.

### Contributions

Introducing a sampling step for regions of interest in the group regions using the Sequential Monte Carlo approach.

Derivation of the likelihood function.

## SMC Framework

$$\mathbf{X}_k^g = \left( \mathbf{x}'_{1,k}, \dots, \mathbf{x}'_{t,k}, \dots, \mathbf{x}'_{n_T^g,k}, \mathbf{G}_k \right)', \text{ where}$$

- $\mathbf{x}_{t,k}$  - the state vector of the  $t^{\text{th}}$  target,  $t = 1, \dots, n_T^g$ , at time  $k$
- $\mathbf{G}_k$  is a vector characterising the group

**The system dynamics is given by:**

$$\mathbf{X}_k^g = f(\mathbf{X}_{k-1}^g, \boldsymbol{\eta}_{k-1}), \text{ where } \boldsymbol{\eta}_k \text{ is the system noise.}$$

**The sensors measurements are described as:**

$$\mathbf{Z}_k = h(\mathbf{X}_k^g, \mathbf{w}_k),$$

where  $\mathbf{w}_k$  is the measurement noise and  $\mathbf{Z}_k = \{\mathbf{z}_{m,k}\}_{m=1}^{M_k}$  is the set of measurements from the objects received at time step  $k$ .

Estimate:  $p(\mathbf{X}_k^g | \mathbf{Z}_{1:k})$



## SMC Framework

The posterior state PDF is estimated given the data

$\mathbf{Z}_{1:k} = \mathbf{Z}_1, \dots, \mathbf{Z}_k$ , in two steps:

- prediction:

$$p(\mathbf{x}_k^g | \mathbf{Z}_{1:k-1}) = \int p(\mathbf{x}_k^g | \mathbf{x}_{k-1}^g) p(\mathbf{x}_{k-1}^g | \mathbf{Z}_{1:k-1}) d\mathbf{x}_{k-1}^g,$$

- update:

$$p(\mathbf{x}_k^g | \mathbf{Z}_{1:k}) = \frac{p(\mathbf{Z}_k | \mathbf{x}_k^g) p(\mathbf{x}_k^g | \mathbf{Z}_{1:k-1})}{p(\mathbf{Z}_k | \mathbf{Z}_{1:k-1})},$$

where  $p(\mathbf{Z}_k | \mathbf{Z}_{1:k-1})$  is the normalising constant.

The number of measurements  $M_{t,k} \sim \text{Poisson}(\lambda_t)$

$$p(\mathbf{Z}_k | \mathbf{x}_{t,k}) = \prod_{m=1}^{M_{t,k}} p(\mathbf{z}_{m,k} | \mathbf{x}_k).$$

They are independent!



## Likelihood Function Based on Surface Parametrisation

Using the Chapman-Kolmogorov equation we introduce the measurement sources  $\mathbf{V}_k \in \mathcal{V}_k(\mathbf{X}_k^g, \mathbf{x}_{s,k}), :$

$$p(\mathbf{z}_{m,k} | \mathbf{X}_k^g) = \int_{\mathbb{R}^{n_v}} p(\mathbf{z}_{m,k} | \mathbf{V}_k) p(\mathbf{V}_k | \mathbf{X}_k^g) d\mathbf{V}_k,$$

where

- $p(\mathbf{z}_{m,k} | \mathbf{V}_k)$  is the probability of receiving the measurement  $\mathbf{z}_{m,k}$  if the actual source of it is  $\mathbf{V}_k$ ;
- $p(\mathbf{V}_k | \mathbf{X}_k^g)$  is the probability of a point in the state space to be a source of measurement given the group object  $\mathbf{X}_k^g$ .

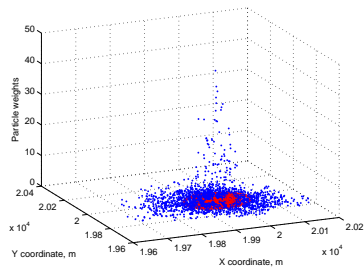
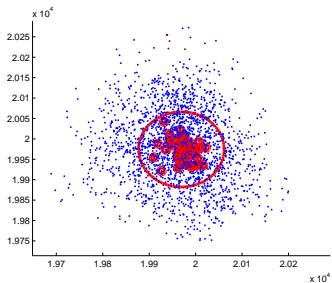
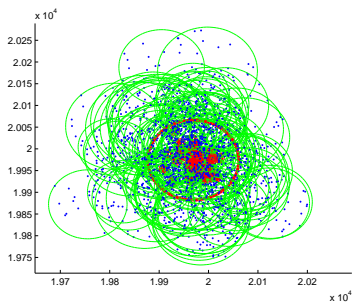
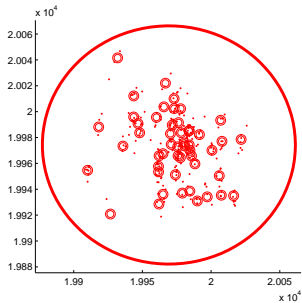
## Parameterisation of the Visible Surface

$$\begin{aligned}
 p(\mathbf{z}_{m,k} | \mathbf{X}_{k|k-1}^{g(i)}) &= \int_{\mathbb{R}^{n_x}} p(\mathbf{z}_{m,k} | \mathbf{V}_k) p(\mathbf{V}_k | \mathbf{X}_{k|k-1}^{g(i)}) d\mathbf{V}_k \\
 &\approx \sum_{\ell=1}^S p(\mathbf{z}_{m,k} | \mathbf{V}_k^{(\ell)}) p(\mathbf{V}_k^{(\ell)} | \mathbf{X}_{k|k-1}^{g(i)}).
 \end{aligned}$$

### For example

$$p(\mathbf{z}_{m,k} | \mathbf{V}_k^{(\ell)}) = \frac{1}{\sqrt{2\pi \|\mathbf{R}\|}} e^{-\frac{(\mathbf{z}_{m,k} - \mathbf{z}_k^{(\ell)})^T \mathbf{R}^{-1} (\mathbf{z}_{m,k} - \mathbf{z}_k^{(\ell)})}{2}};$$

$$p(\mathbf{V}_k^{(\ell)} | \mathbf{X}_{k|k-1}^{g(i)}) = \mathcal{U}_{\mathcal{C}(x_{c,k}^{(i)}, y_{c,k}^{(i)}, r_{k|k-1}^{(i)})} \left( \sqrt{(x_k^{(\ell)} - x_{c,k}^{(i)})^2 + (y_k^{(\ell)} - y_{c,k}^{(i)})^2} \right).$$



## Scenario

- one group of objects
- nearly constant velocity motion model for the individual targets;
- circular shape surrounding the group;
- range and bearing measurements;
- multiple sensors observe the objects in the group
- the number of measurements is  $\sim$  Poisson(5);
- 200 time steps each repeated for 30 iterations;
- 100 group object particles;
- 20 samples per particle;





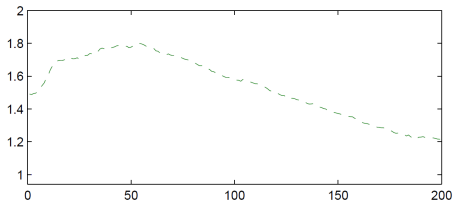
## Results (video)

(Loading GroupExample.avi)

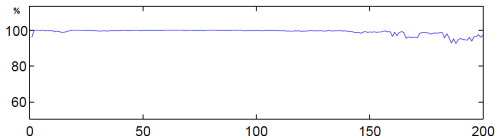


## Results

Performance evaluation averaged over 30 runs:



Ratio between the estimated extent and the optimal extent



Percentage of targets inside the group extent

## Summary

- In this paper we cope with the problem of having *multiple measurements* from a large number of objects with coordinated *group movement* by deriving an expression for the *likelihood function* based on *surface parametrisation*.
- The algorithm is presented in a general framework using *nonlinear measurements* and *nonlinear system model* as well as *noise with arbitrary distribution*.
- We show how the data *association problem* could be *facilitated* using the likelihood representation.

## Future Work

- More complex target shapes (i.e. ellipse, freeform shapes)
- Better sampling in the group region
- Variable number of particle/samples, use of box particles
- Clutter scenarios
- Multiple groups and interactions between them
- Estimation of the number of targets

Thank you for your attention!

