Advanced Statistical Strategy for Generation of Non-Normally distributed PSP Compact Model Parameters and Statistical Circuit Simulation

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Summary

- Background
- Physical simulation
- Compact model extraction
- Principle Component Analysis
- Nonlinear Power Method
- Conclusions

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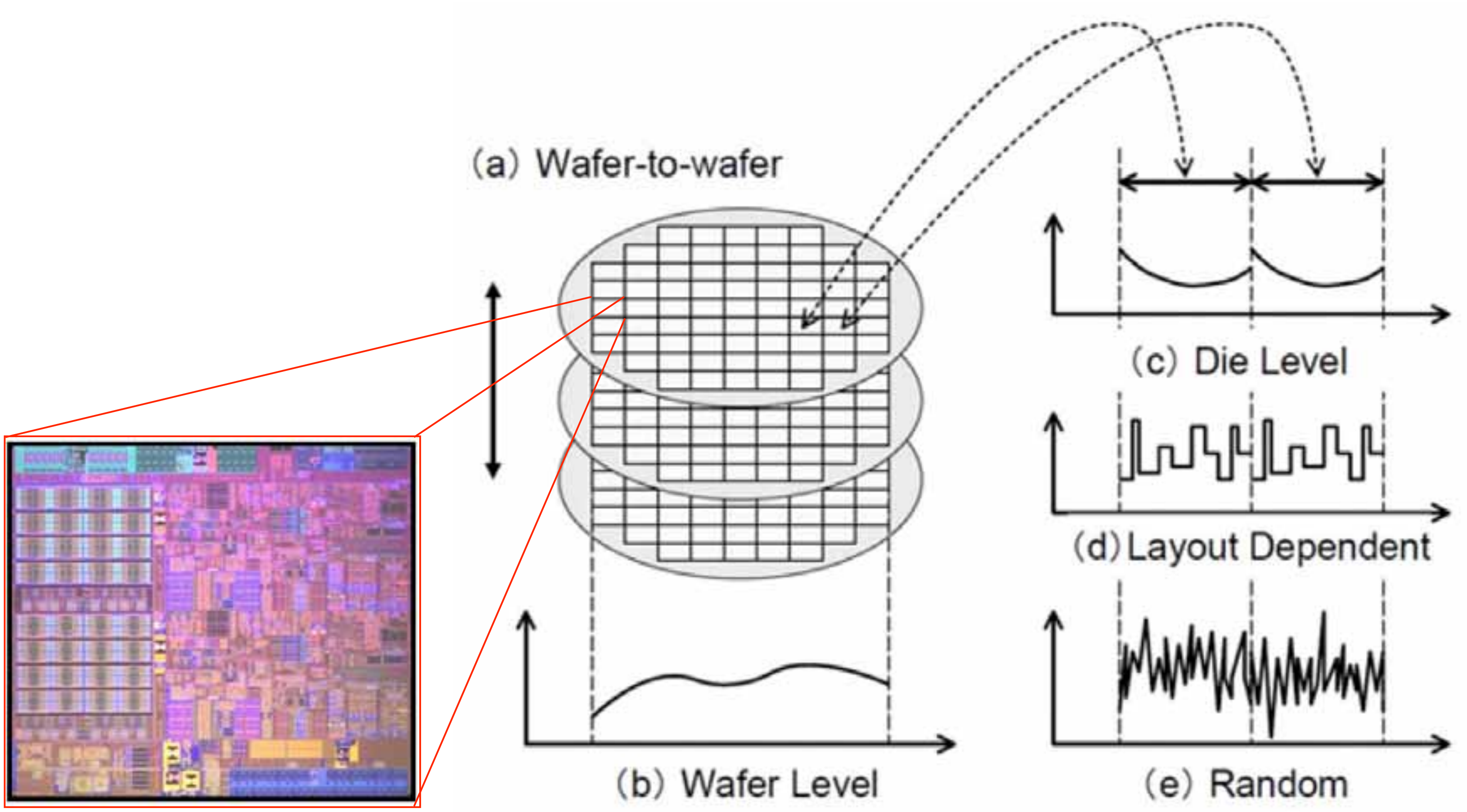
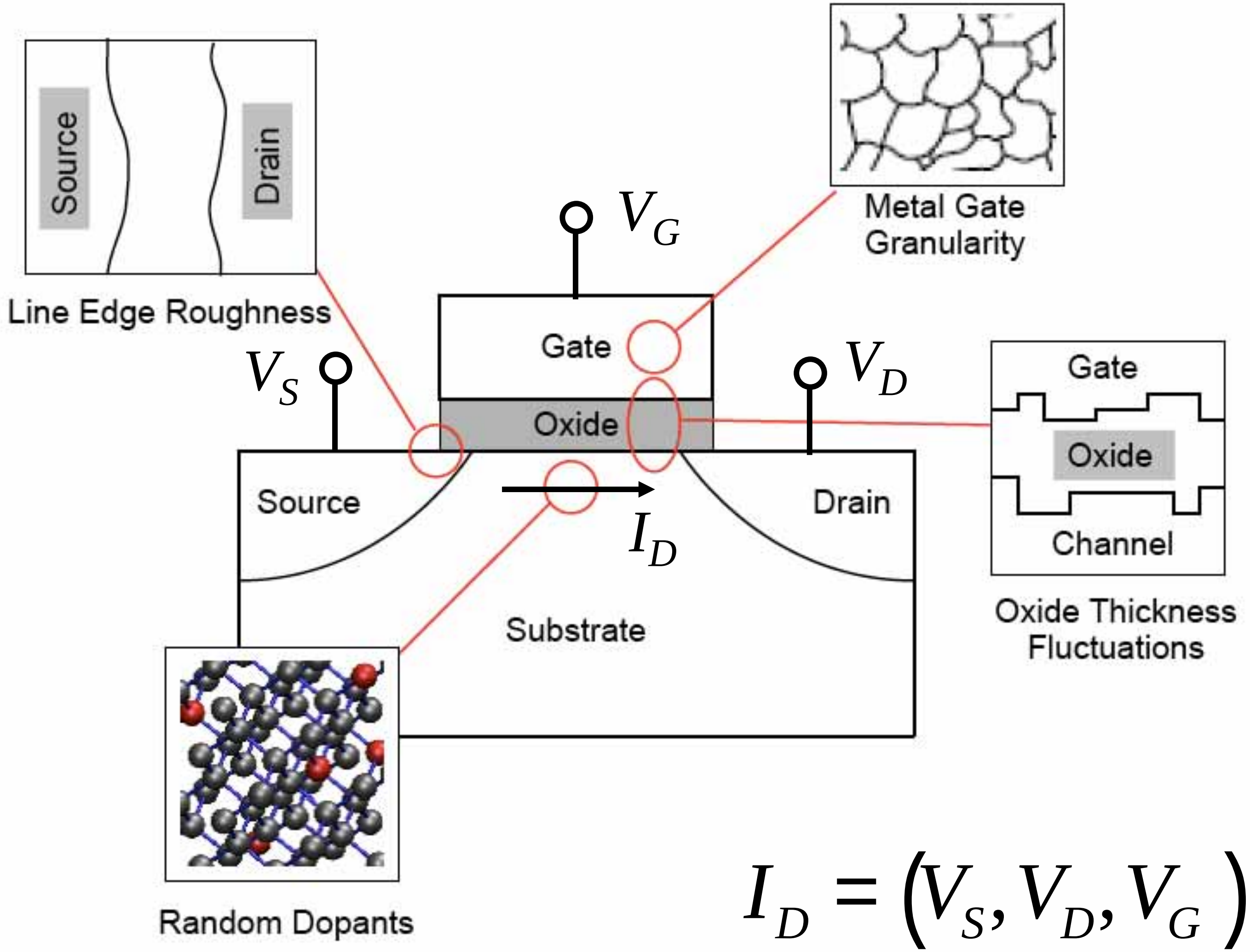
CMOS variability classification



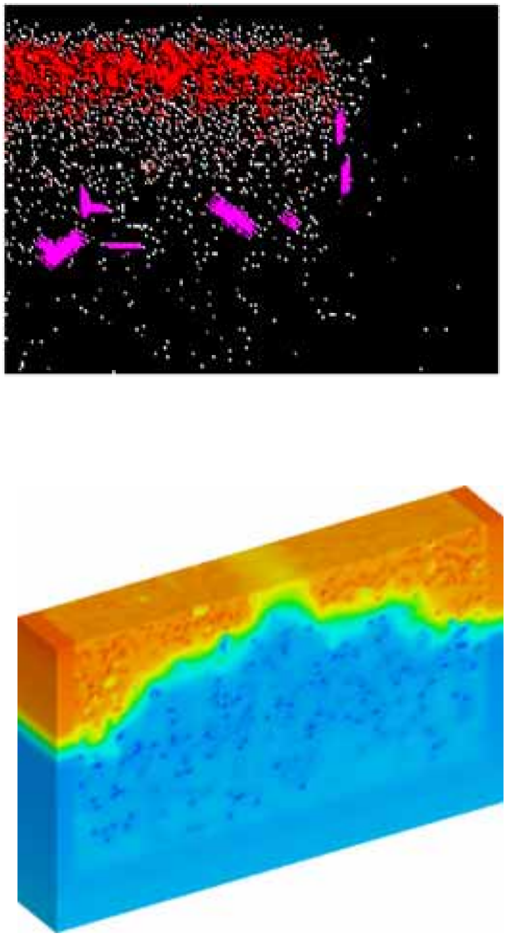
After K. Takeuchi (NEC)

Sources of statistical variability

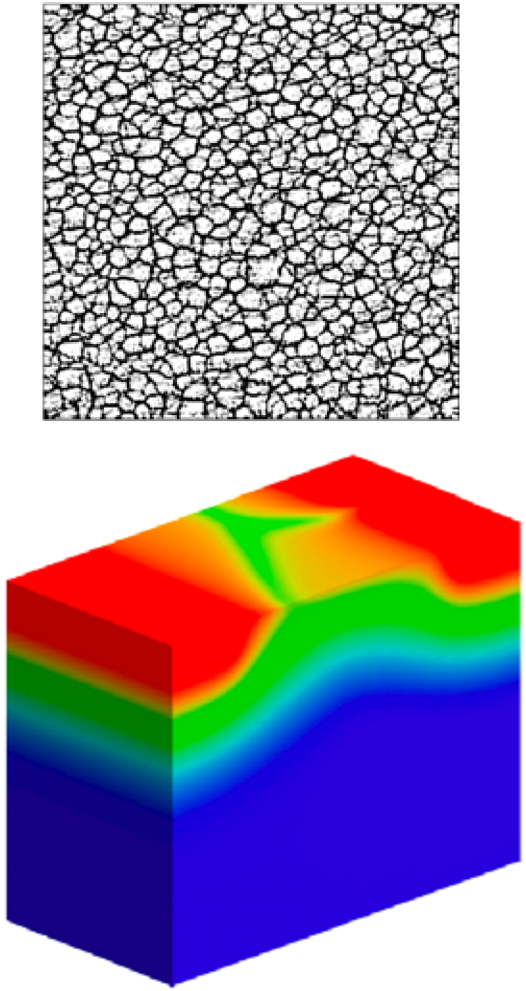


$$I_D = (V_S, V_D, V_G)$$

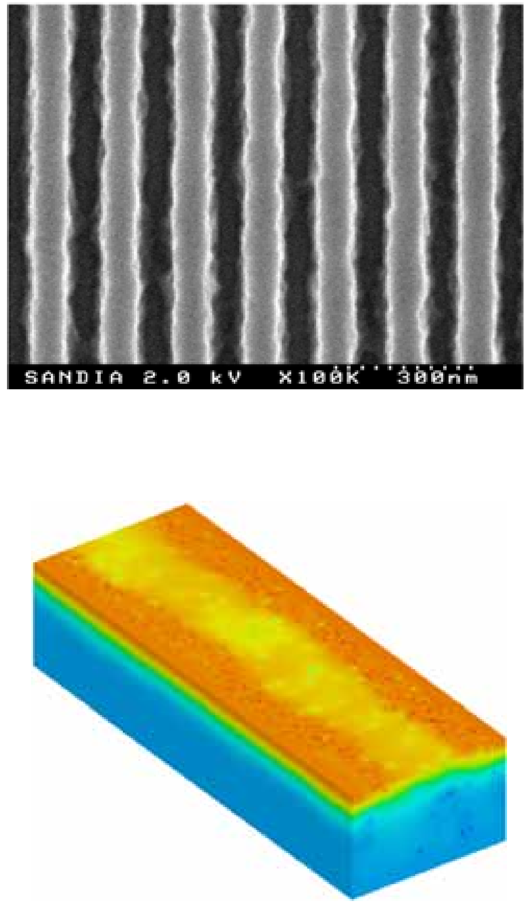
Sources of statistical variability



Random dopants



Polysilicon/high-k Granularity



Line edge roughness

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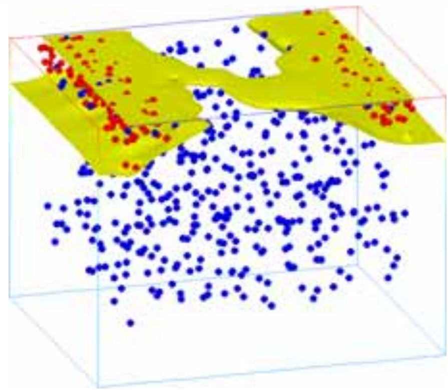
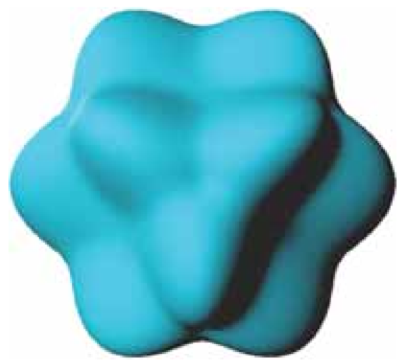
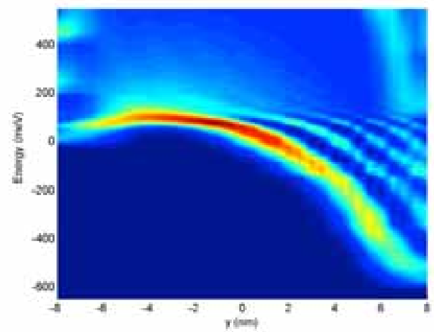
GSS ‘atomistic’ simulation tools

- ❑3D DD simulator
  - Random discrete dopants
  - Random interface roughness
  - Line edge roughness
  - DG quantum corrections
- ❑3D MC simulator
  - Si/S-Si/SiGe/III-V
  - New interface scattering models
  - Degeneracy
  - High-*k* dielectrics
  - *Ab-initio* impurity scattering
  - *Ab-initio* interface roughness
- ❑3D NEGF simulator
  - Full 3D NEGF
  - Coupled mode space 3D NEGF
  - Includes scattering







The basic semiconductor equations

The basic equations that describe the operation of most semiconductor devices are:

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\varepsilon_{Si}}\left[p(x) - n(x) + N_D^+(x) - N_A^-(x)\right]$$

Poisson's equation

$$\frac{dn}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} - R_n + G_n$$

$$\frac{dp}{dt} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - R_p + G_p$$

The continuity equations for electrons and holes based on conservation of mobile charge.

Where

$$J_n = -qn\mu_n\left(\frac{d\psi}{dx} - \frac{k_BT}{qn}\frac{dn}{dx}\right) = -qn\mu_n\frac{d\phi_n}{dx}$$

$$J_p = -qp\mu_p\left(\frac{d\psi}{dx} + \frac{k_BT}{qp}\frac{dp}{dx}\right) = -qp\mu_p\frac{d\phi_p}{dx}$$

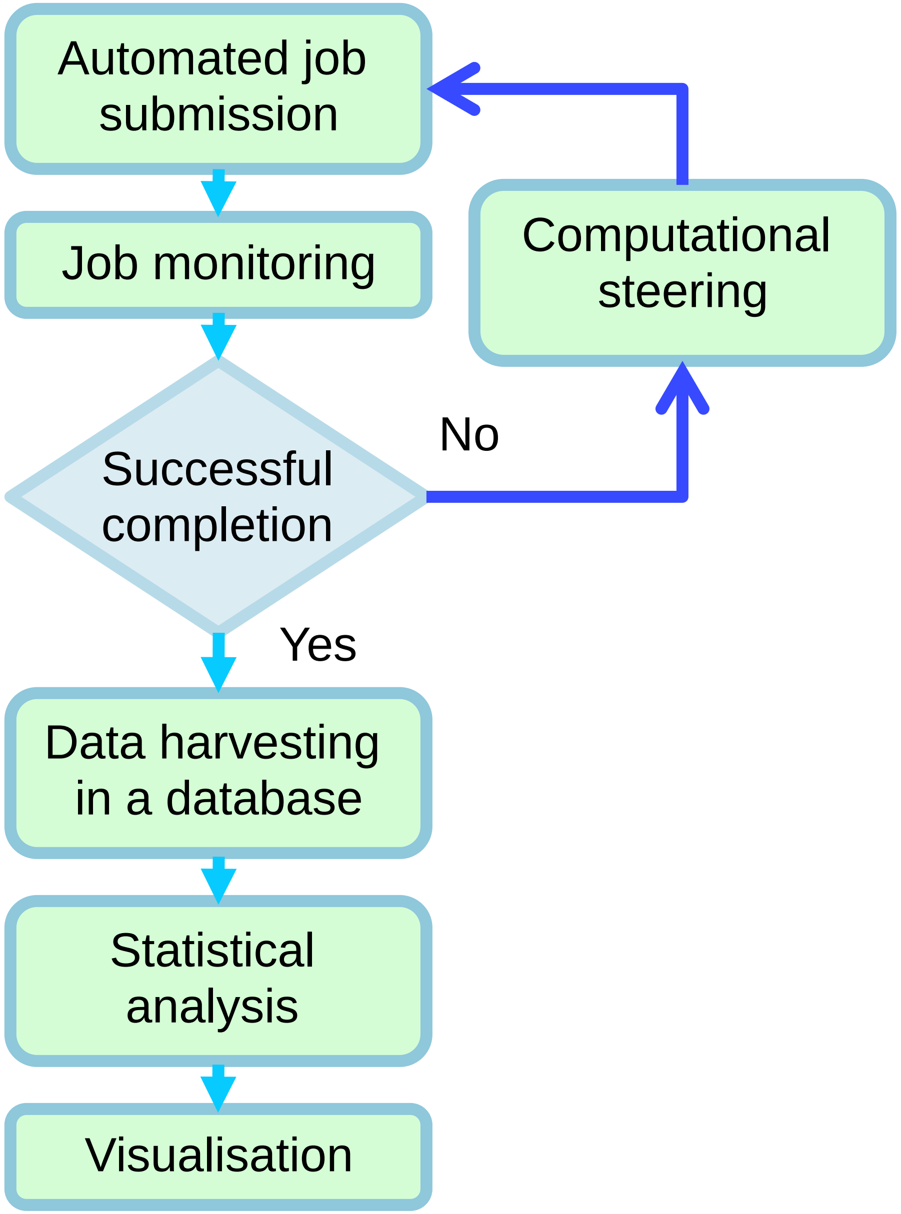
ϕ_n, ϕ_p quasi-Fermi potentials

$$\phi_n = \psi - \frac{k_BT}{q}\ln\left(\frac{n}{n_i}\right)$$

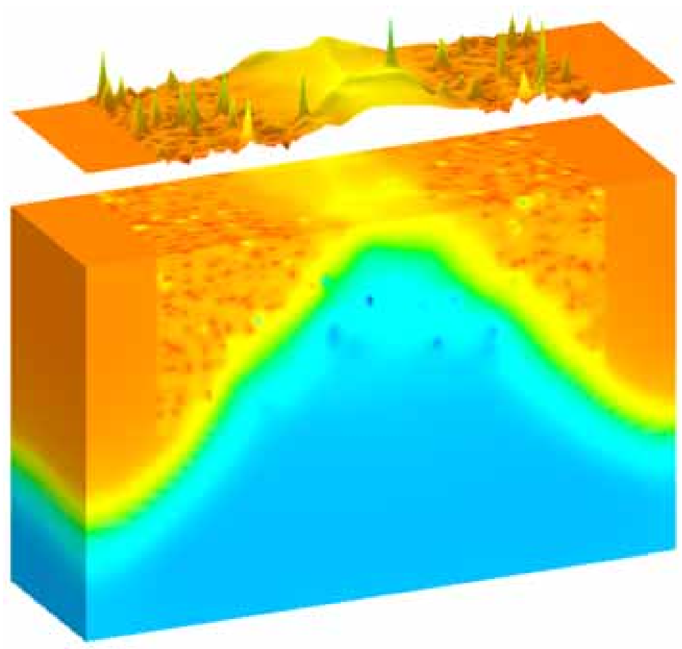
$$\phi_p = \psi + \frac{k_BT}{q}\ln\left(\frac{p}{n_i}\right)$$

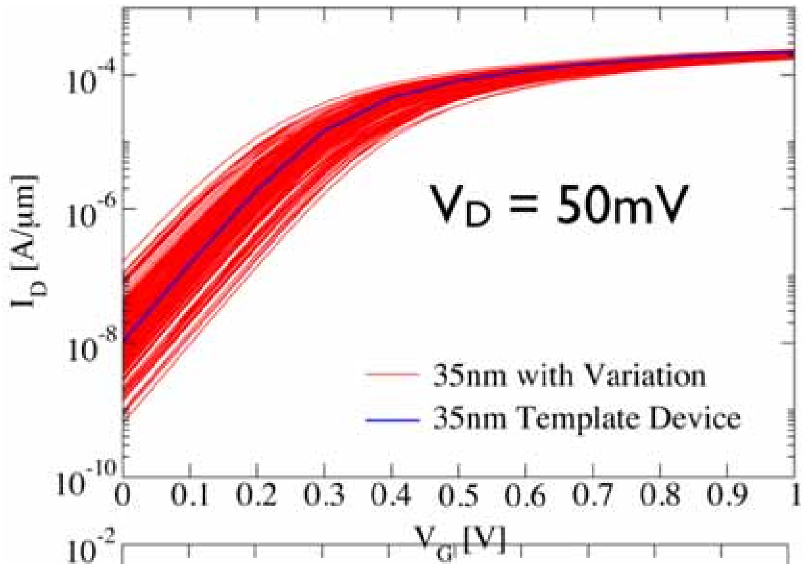
Grid/cluster based simulation technology

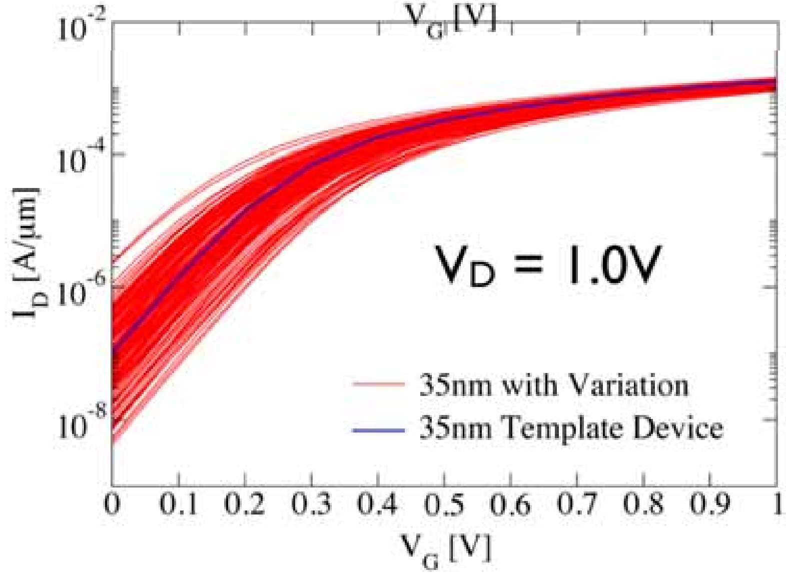
2380 CPUs



Statistical simulation results
35 nm MOSFET







RDD+LER+PSG
Compact models

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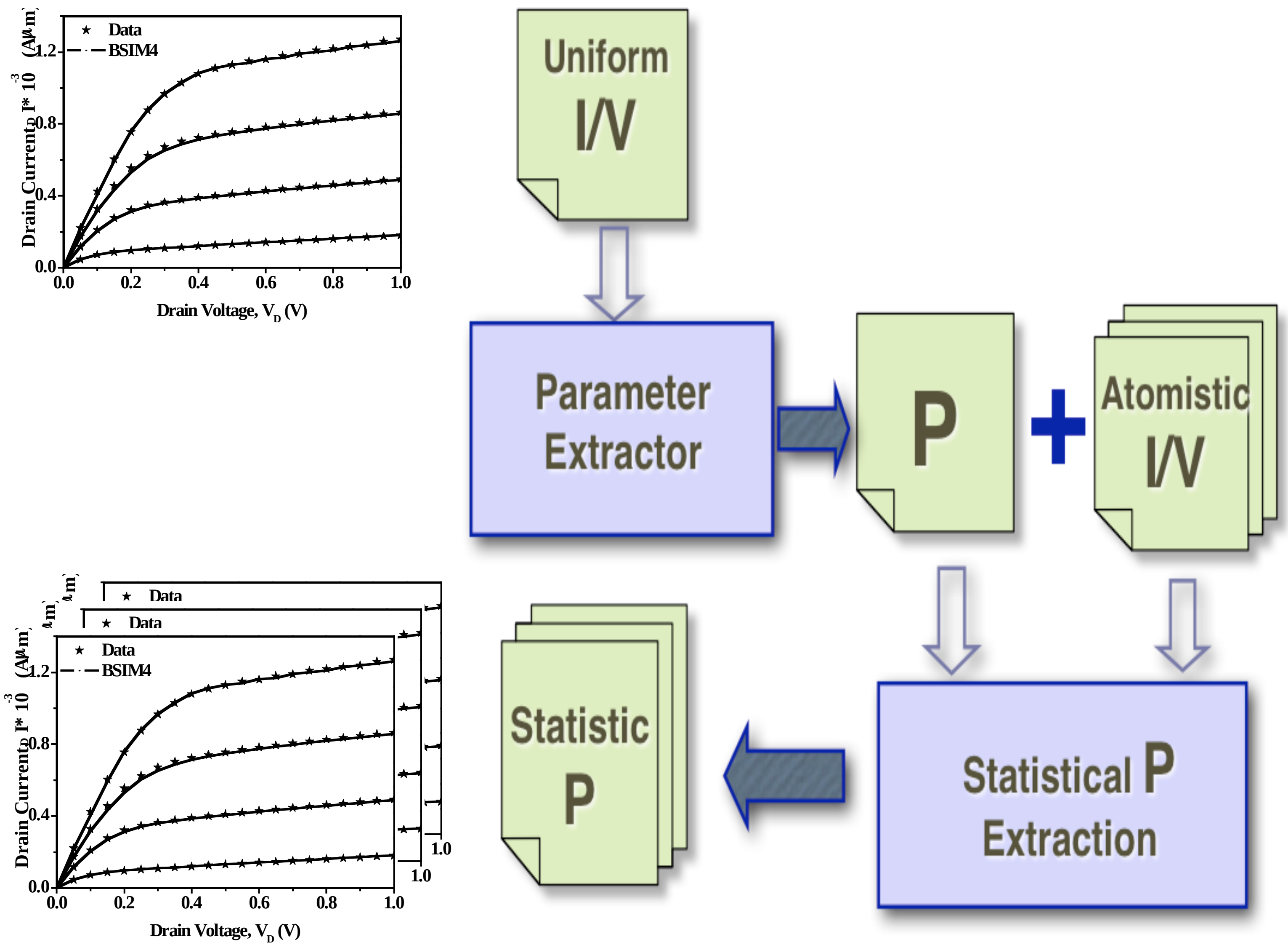
Compact models

- Compact models (CM) used in circuit simulators like SPICE are the interface between technology and design.
- CM are usually closed form analytical expressions returning terminal currents as a function of applied bias.
- CM have a large number of parameters determined by fitting to measured or simulated transistor characteristics.
- The industrial standard compact models are BSIM and PSP.

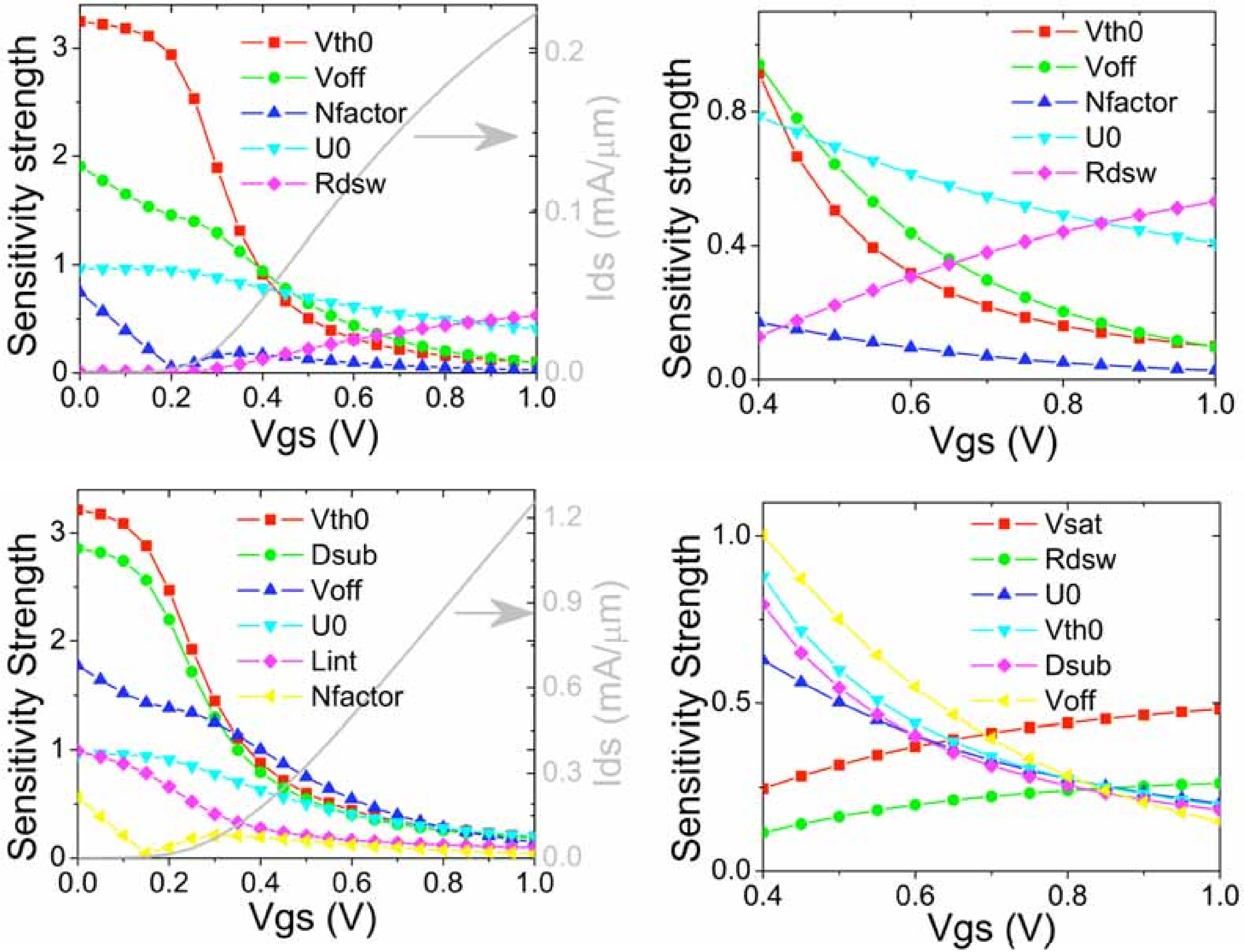
$$I_D = \left(V_S, V_D, V_G, p1, p2, \ldots pn\right)$$

Two stage parameter extraction

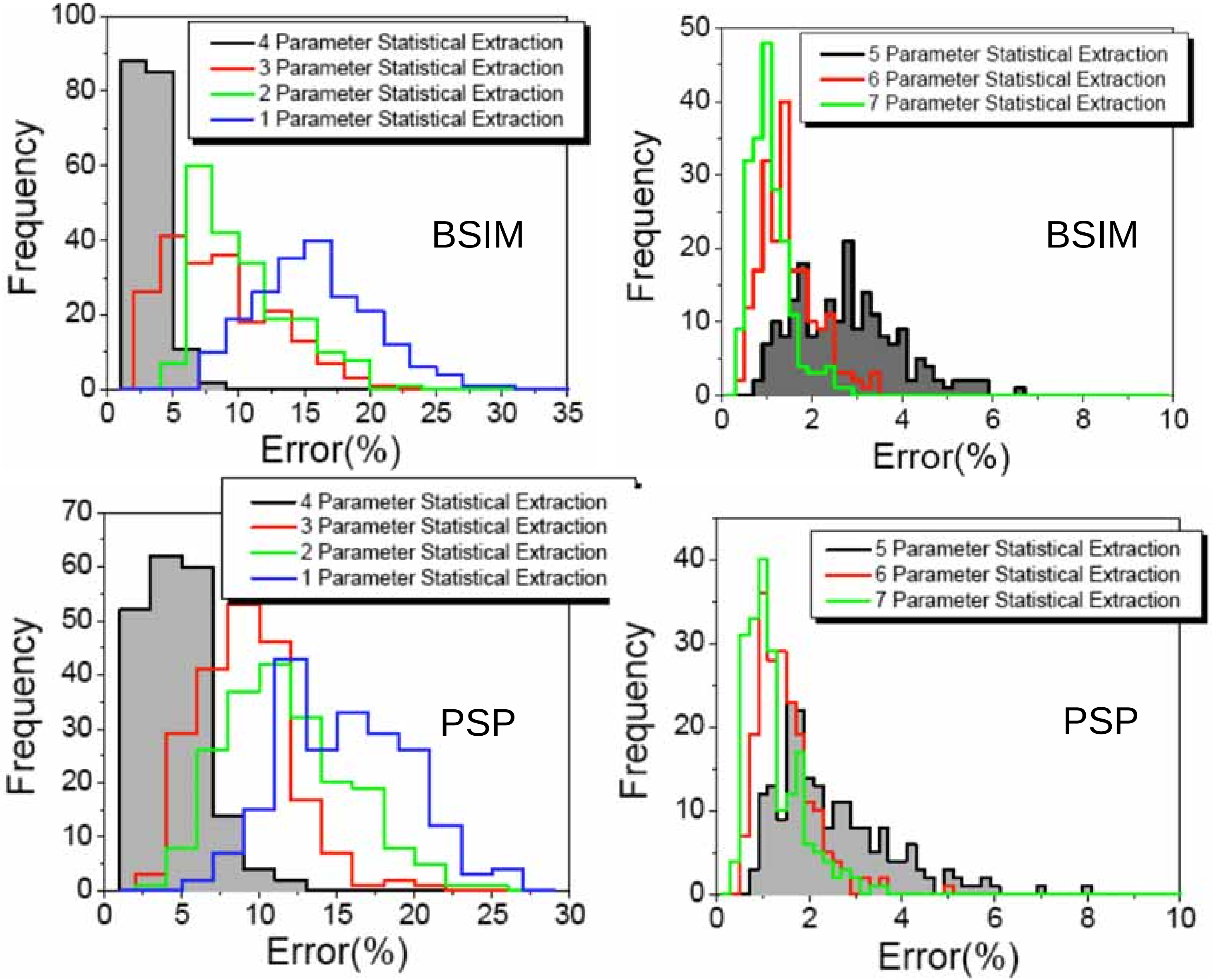


Large set of microscopically different transistors

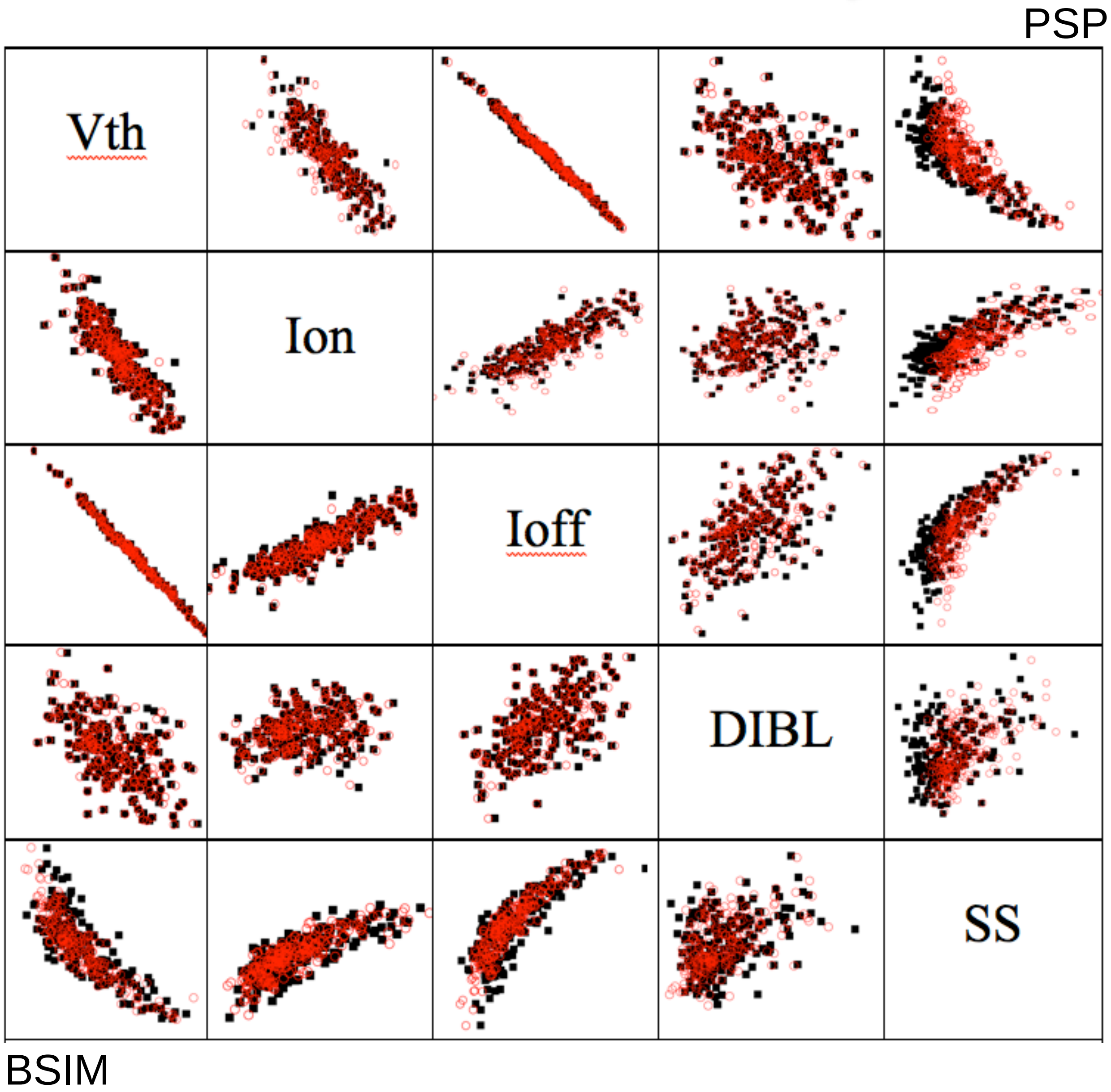
Comprehensive sensitivity analysis



Statistical accuracy



Statistical accuracy



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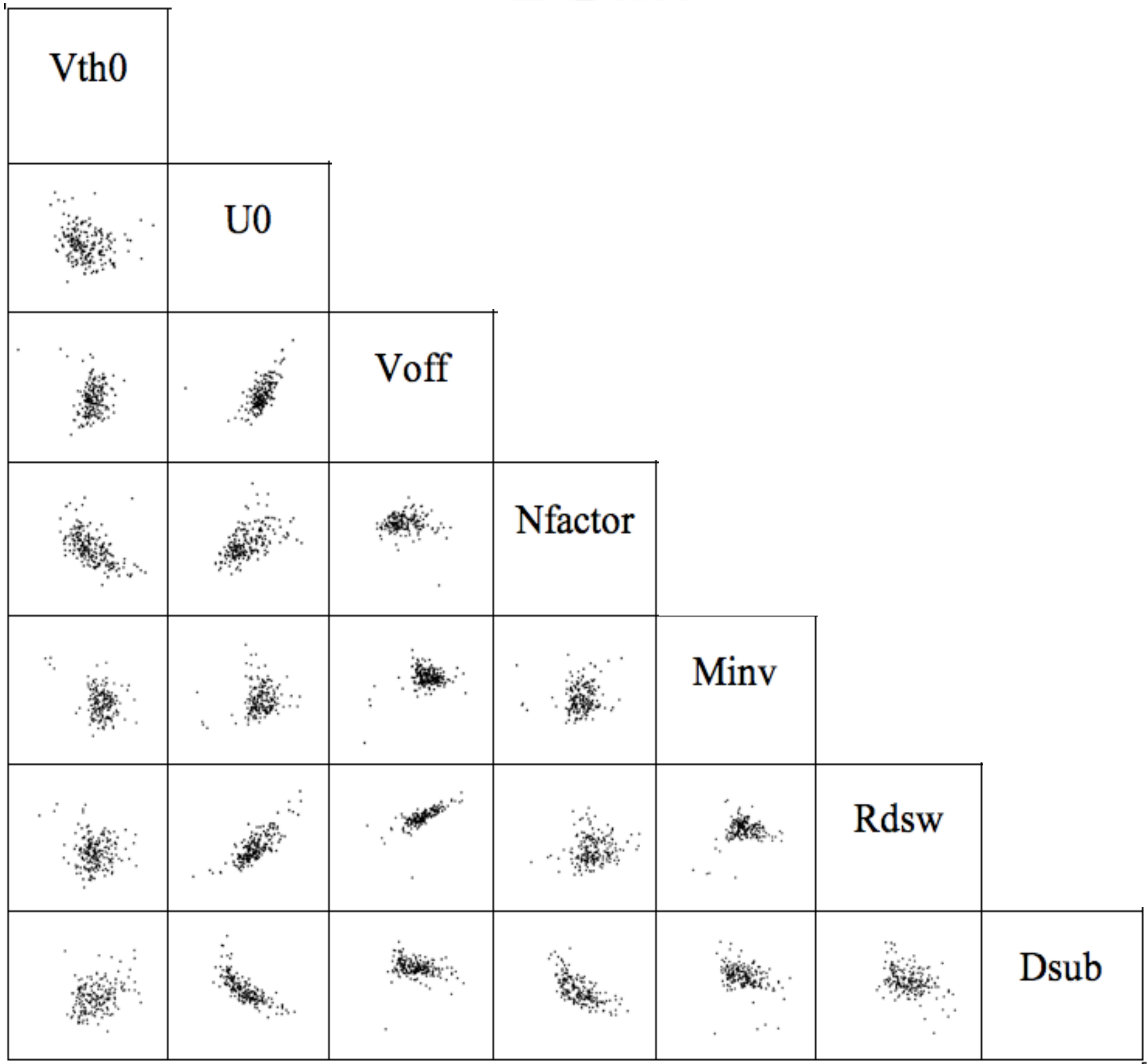
Statistical compact model parameter correlations BSIM



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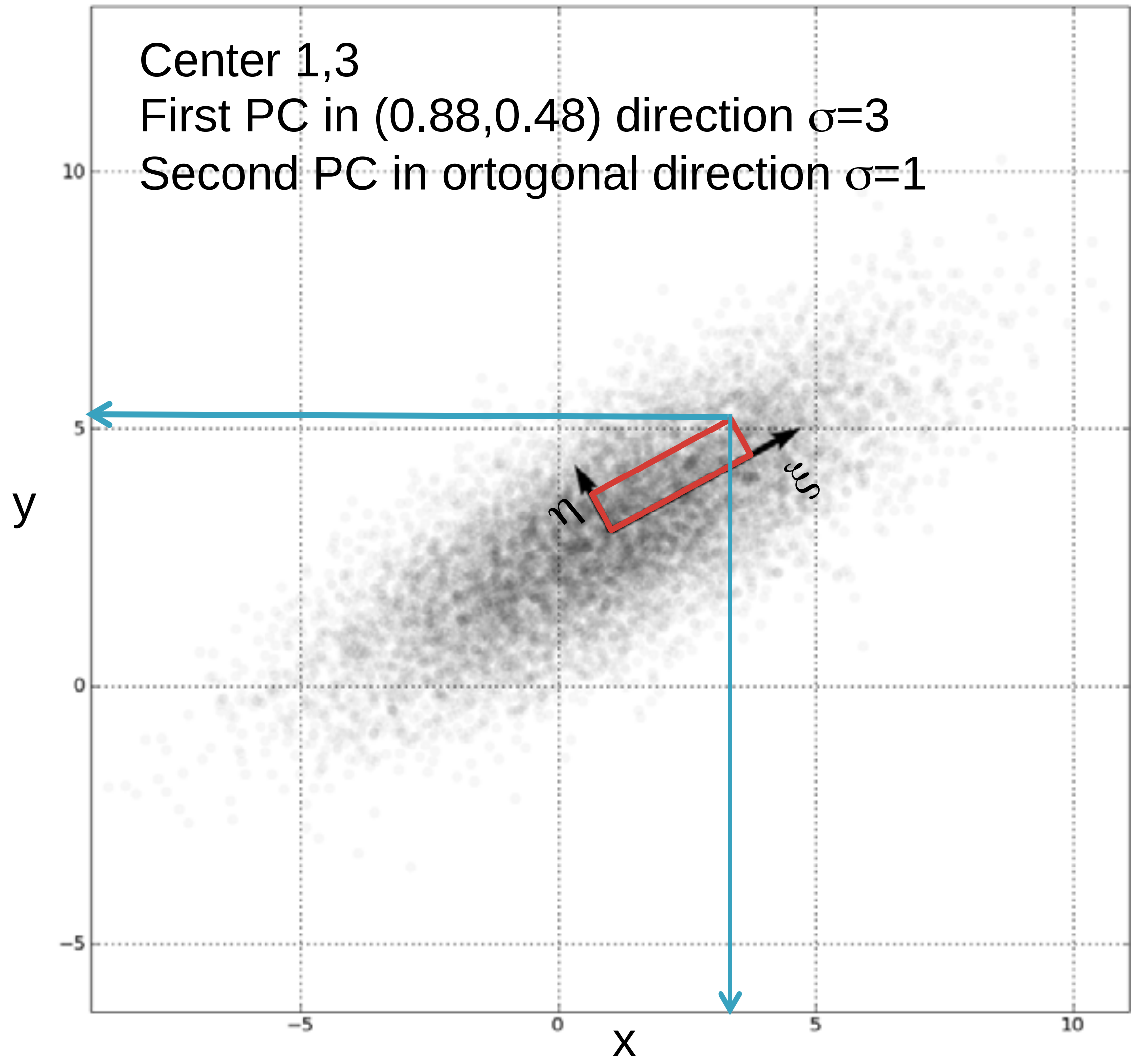
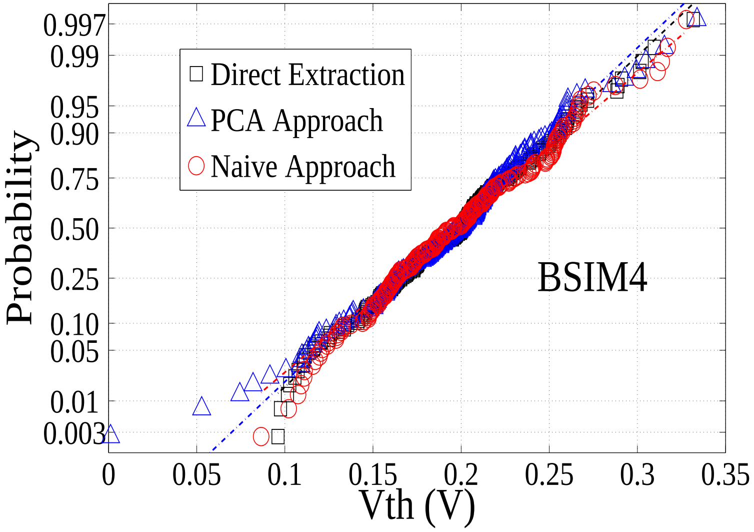
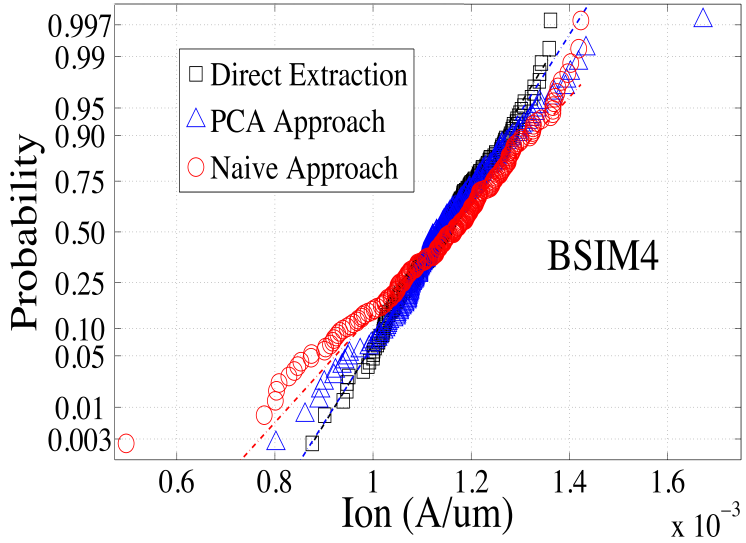
PCA

PCA converts a set of observations of correlated variables into a set of values of uncorrelated variables called Principle components.

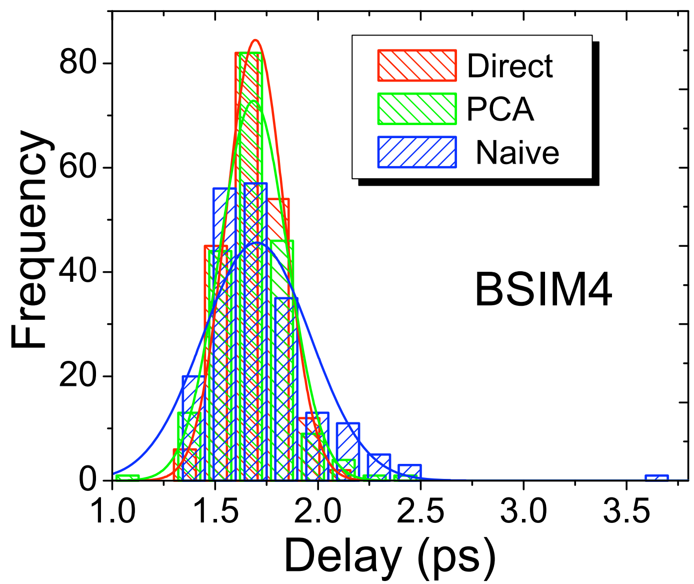


Naïve approach vs. PCA

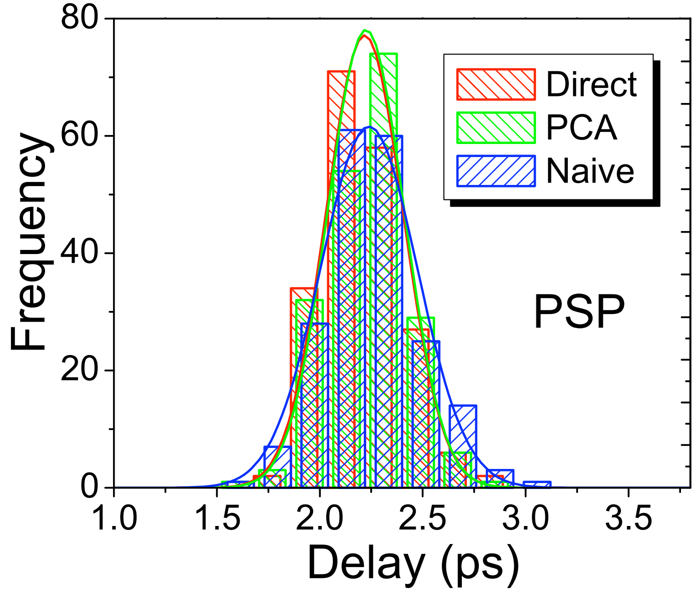




Naïve 85%, PCA 15%



Naïve 25%, PCA 5%



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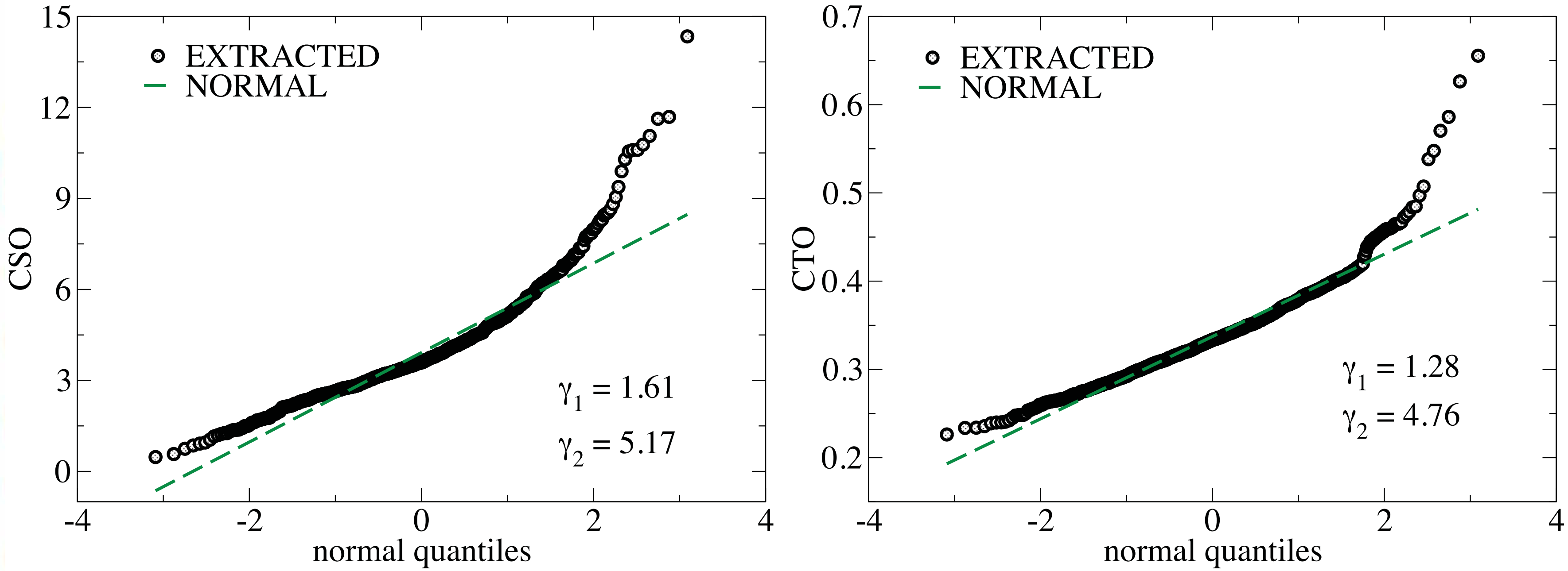
The compact model parameters are not normally distributed



The Nonlinear Power Method (NPM)

- The NPM preserves the correlations and reproduces the higher moments of the SCM parameter distributions
- The NPM generates multivariate non-normal distributions with an arbitrary covariance matrix from a set of analytical equations

The Nonlinear Power Method (NPM)

$$Y_i = \mathbf{c}_i^T \mathbf{Z}_i$$

$E[Y_i] = \mathbf{c}_i^T E[\mathbf{Z}_i]$ Average

$VAR[Y_i] = E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^2\right] - \left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right)^2$ Variance

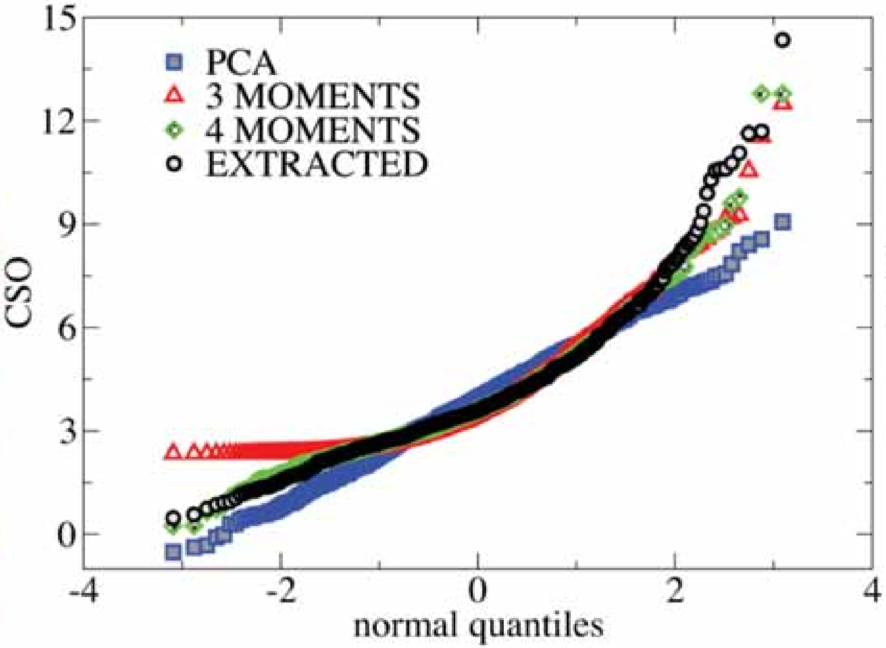
$$\gamma_{1i} = \frac{E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^3\right] - 3E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^2\right]\left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right) + 2\left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right)^3}{\left(VAR[Y_i]\right)^{3/2}}$$

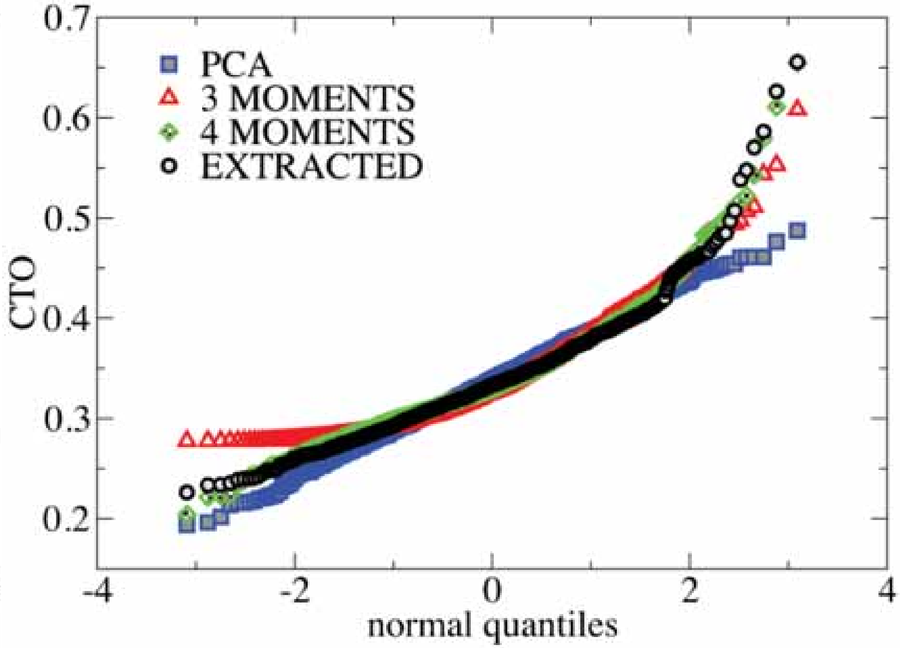
Skew

$$\gamma_{2i} = \frac{E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^4\right] - 4E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^3\right]\left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right) - 3\left(E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^2\right]\right)^2 + 12E\left[\left(\mathbf{c}_i^T \mathbf{Z}_i\right)^2\right]\left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right)^2 + 6\left(E\left[\mathbf{c}_i^T \mathbf{Z}_i\right]\right)^4}{\left(VAR[Y_i]\right)^2}$$

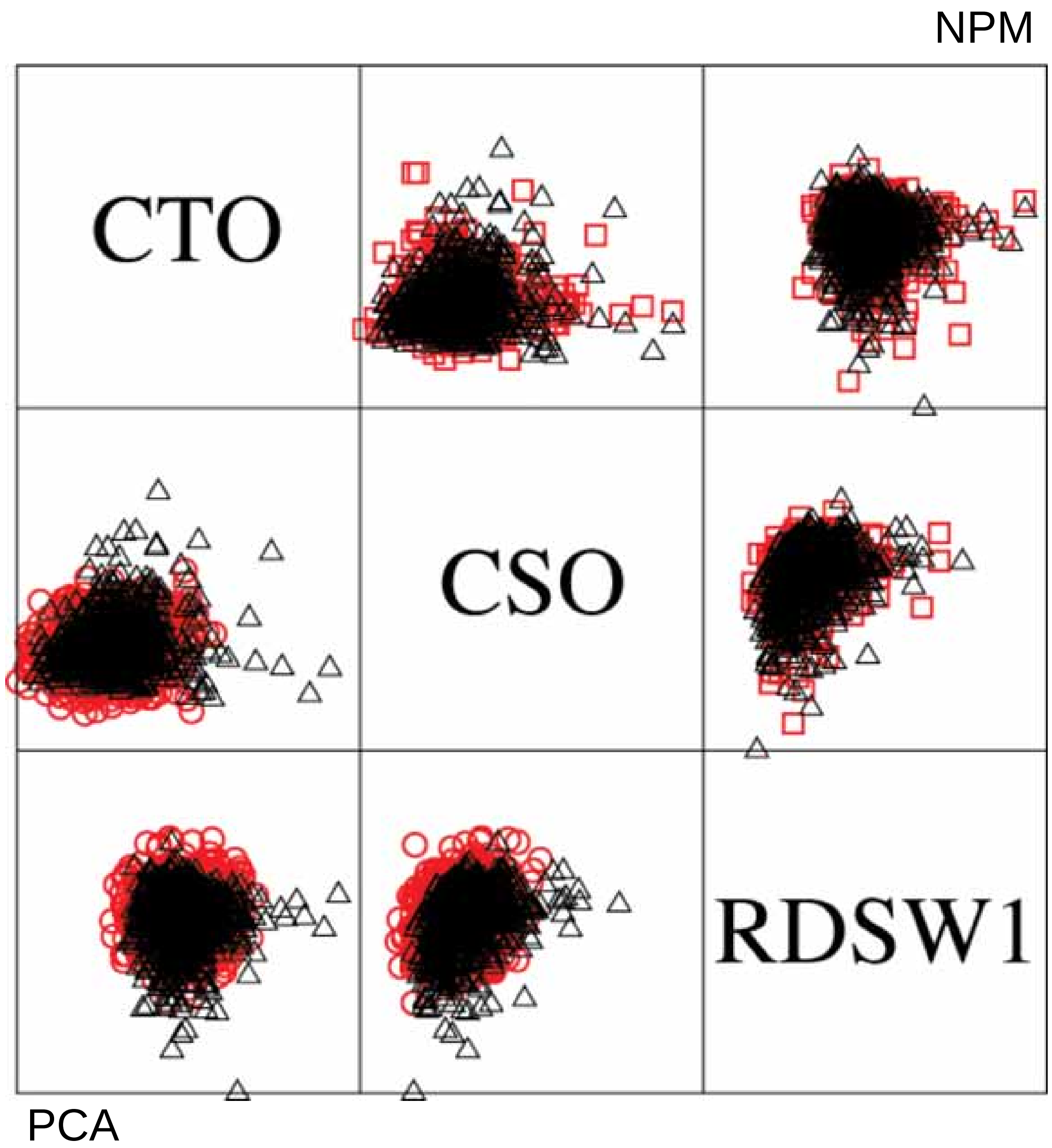
Kurtosis

The nonlinear power method (NPM) can cope with non normal distributions

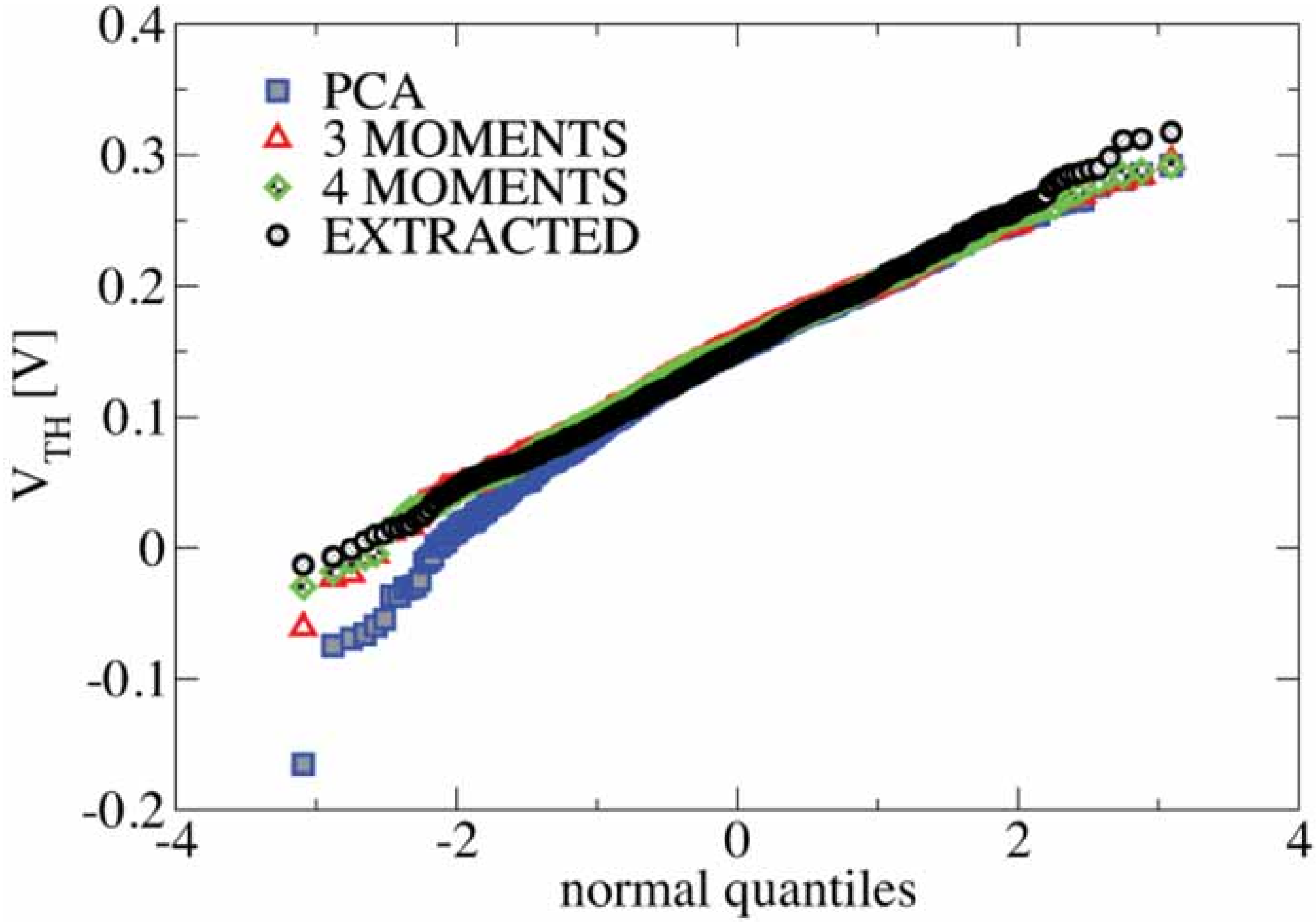




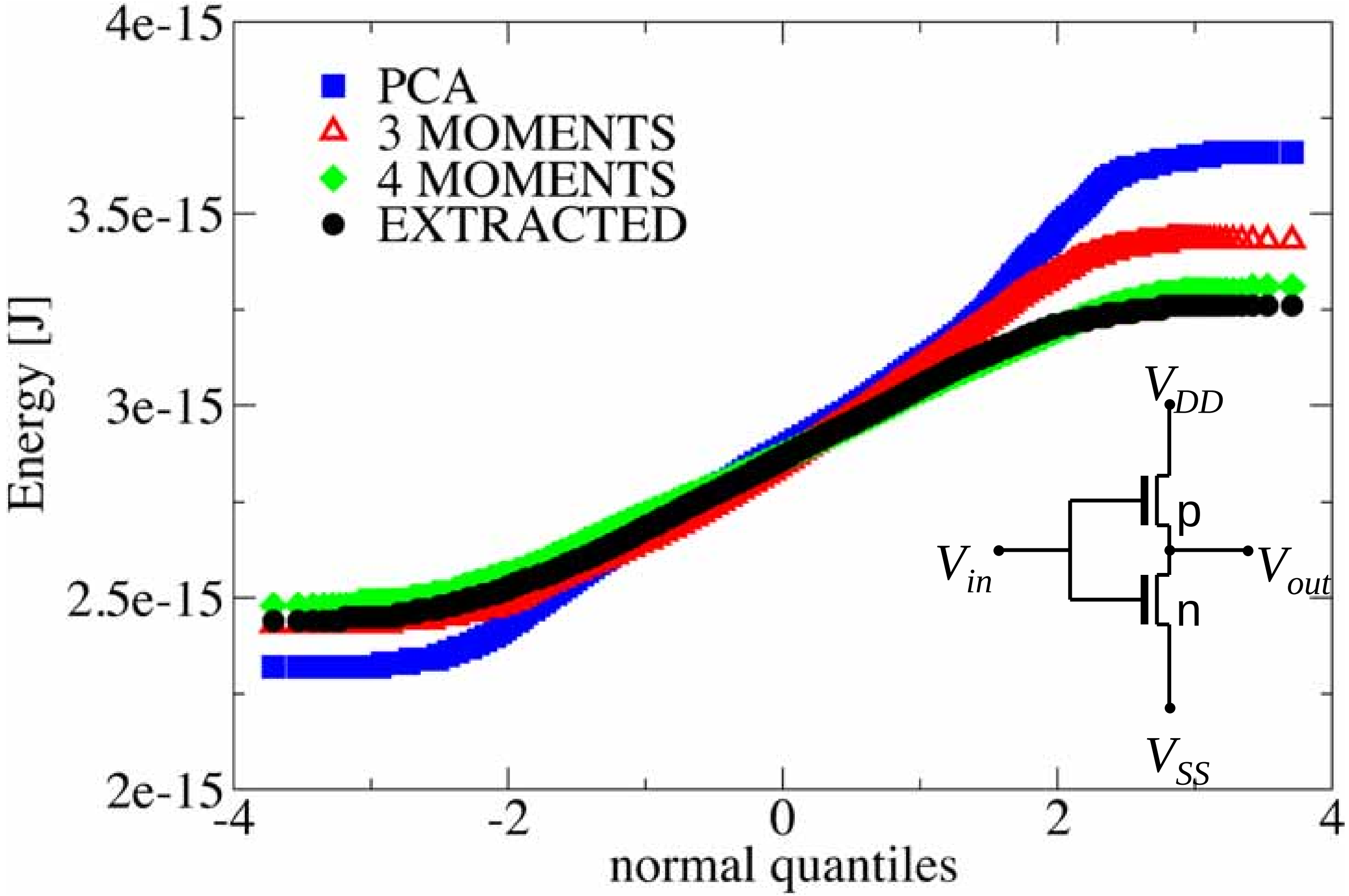
NPM can cope also with the correlations



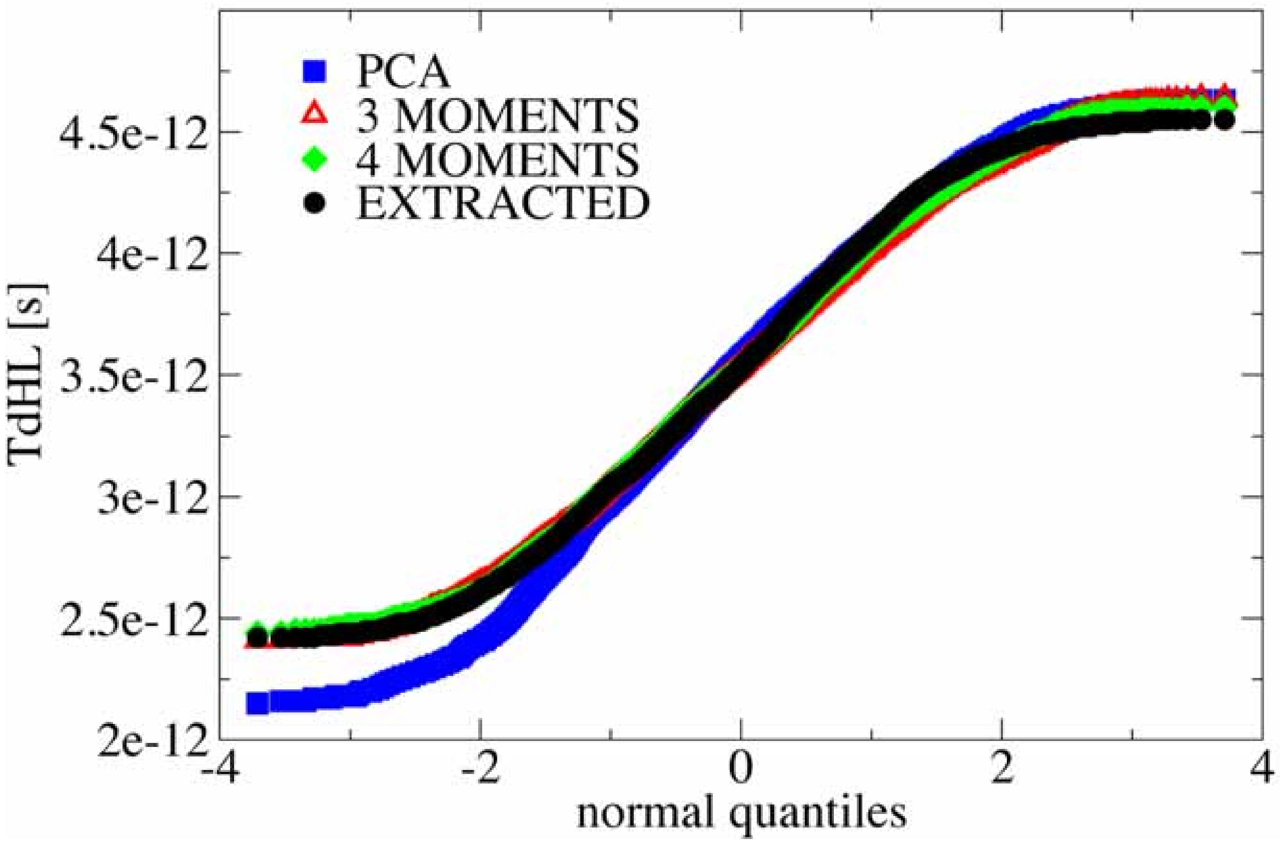
NPM reproducers the distribution of important figures of merit



Energy distribution of an invertor



Timing distribution in an invertor



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Conclusions

- The statistical compact model parameters are correlated.
- The distribution of the individual parameters deviate from normal.
- PCA fails to reproduce the proper distribution and correlation of the statistical compact model parameters.
- NPM not only accurately reproduces the accurately the parameters distribution and correlations but transistor figures of merit and circuit simulation results.