

# MMAS and ACS for GPS Surveying Problem

STEFKA FIDANOVA  
Bulgarian Academy of Sciences  
Institute for Parallel Processing  
Acad G. Bonchev Street 25A, 1113 Sofia  
BULGARIA  
stefka@parallel.bas.bg

*Abstract:* Ant Colony Optimization(ACO) have been used successfully to solve hard combinatorial optimization problems. This metaheuristics method is inspired by the foraging behavior of ant colonies, which manage to establish the shortest routes between their colonies to feeding sources and back. In this paper ACO algorithms are developed to provide near-optimal solutions for Global Positioning System surveying problem. In designing Global Positioning System (GPS) surveying network, a given set of earth points must be observed consecutively (schedule). The cost of the schedule is the sum of the time needed to go from one point to another. The problem is to search for the best order in which this observation is executed. Minimizing the cost of this schedule is the goal of this work.

*Key-Words:* Ant Colony Optimization, metaheuristics, GPS surveying

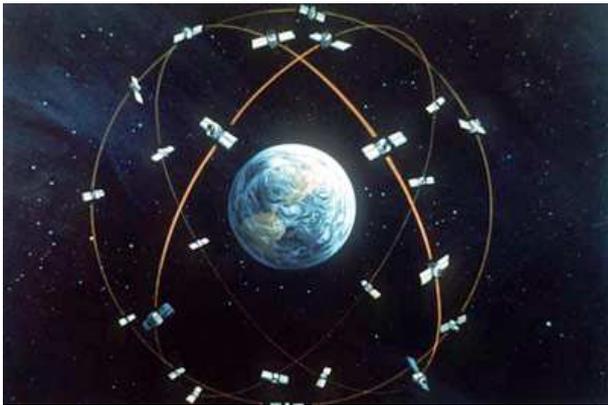


Figure 1: GPS satellite system

## 1 Introduction

Satellite navigation systems have an impact in geoscience, in particular on surveying work in quick and effective determining positions and changes in positions networks. The most widely known space systems are: the American NAVSTAR global positioning system, the Russian GLObal Navigation Satellite System (GLONASS), and the forthcoming European satellite navigation system (GALILEO).

GPS satellites continuously transmit radio signals to the Earth while orbiting it (Figure 1). A receiver, with unknown position on Earth, has to detect and converts the signals received from all of the satellites into useful measurements. These measurements

would allow a user to compute a three-dimensional coordinate position: location of the receiver.

Solving this problem to optimality requires a very high computational time. Therefore, metaheuristic methods are used to provide near-optimal solutions for large networks within acceptable amount of computational effort [4, 6, 10]. In this paper we implement two variants of Ant Colony Optimization (ACO) algorithms, Ant Colony System (ACS) and Max-Min Ant System (MMAS). Our aim is to compare there two algorithms for this difficult problem.

The rest of the paper is organized es follows. Section 2 gives an overview of the nature of the problem and its importance. The general framework for a GPS surveying network problem as a combinatorial optimization problem is described in Section 3. Then, ACS and MMAS algorithms are applied in Section 4. The numerical results are presented and discussed in Section 5. The paper ends with conclusions and directions for future research.

## 2 Background

GPS has a strong impact on the art and practice of most forms of positioning and navigation. However, GPS has already had a tremendous impact on surveying, initially as a technology for geodetic surveys [7]. The GPS navigation supports the safe passage of a vessel, aircraft or vehicle from the departure, while underway to its point of arrival; while GPS surveying is mostly associated with the traditional functions

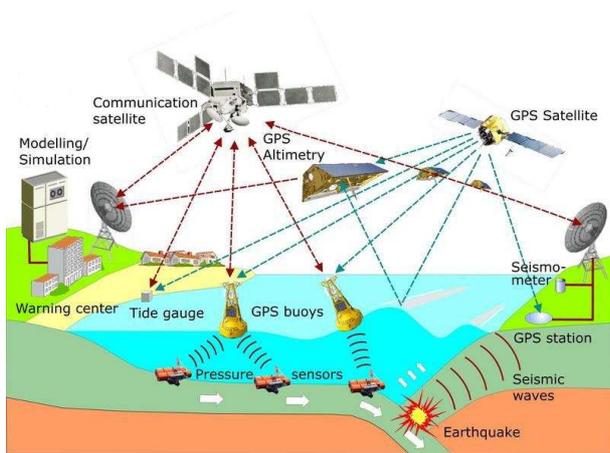


Figure 2: GPS surveying system

of establishing geodetic control, supporting engineering constructions, cadastral surveys and map making. Any GPS observation is proven to have biases, hence, in order to survey an appropriate combination of measurement and processing strategies must be used to minimize their effect on the positioning results. Differencing data collected simultaneously from two or more GPS receivers to several GPS satellites allows to eliminate or significantly reduce most of the biases. All position results are therefore expressed relative to datum stations. GPS relative technology can, in practice, be employed for a wide range of activities and it is competitive against conventional terrestrial technologies of surveying.

Some main uses of GPS surveying will be mentioned:

- **Geodetic Surveys:** GPS has already replaced other methods for establishing, maintaining and densifying geodetic networks. Over distances of a few hundred kilometers, multi stations method gave relative position accuracies of a few decimeters. Unlike it GPS can give accuracies of 1ppm even over distances as short as a few kilometers. Furthermore, GPS is much more faster [7].
- **Scientific Survey:** The measurement of crystal deformation is central to our understanding of earthquake processes, plate motion, ruffing, mountain building mechanism and the near-surface behavior of volcanoes. In this case are measured changes in position, displacement or strain with time. Hence one seeks to repeat the measurements under as nearly an identical set of circumstances and as high an accuracy as possible (few centimeters) [7]. On Figure 2 is shown surveying system for earthquake forecasting.

The GPS positioning technology is superior to the conventional and theodolite procedures, and it can be used by users with no previous experience in satellite surveying.

### 3 Problem Description

The GPS network can be defined as set of stations  $(a_1, a_2, \dots, a_n)$ , which are co-ordinated by placing receivers  $(X1, X2, \dots)$  on them to determine sessions  $(a_1a_2, a_1a_3, a_2a_3, \dots)$  among them. The problem is to search for the best order in which these sessions can be organized to give the best schedule. Thus, the schedule can be defined as a sequence of sessions to be observed consecutively. The solution is represented by linear graph with weighted edges. The nodes represent the stations and the edges represent the moving cost. The objective function of the problem is the cost of the solution which is the sum of the costs (time) to move from one point to another one,  $C(V) = \sum C(a_i, a_j)$ , where  $a_i a_j$  is a session in solution  $V$ . For example if the number of points (stations) is 4, a possible solution is  $V = (a_1, a_3, a_2, a_4)$  and it can be represented by linear graph  $a_1 \rightarrow a_3 \rightarrow a_2 \rightarrow a_4$ . The moving costs are as follows:  $C(a_1, a_3), C(a_3, a_2), C(a_2, a_4)$ . Thus the cost of the solution is  $C(V) = C(a_1, a_3) + C(a_3, a_2) + C(a_2, a_4)$ . In practice, determining how each GPS receiver should be moved between stations to be surveyed in an efficient manner taking into account some important factors such as time, cost etc. The problem is to search for the best order, with respect to the time, in which these sessions can be observed to give the cheapest schedule or to minimize  $C(V)$ . The initial data is a cost matrix, which represents the cost of moving a receiver from one point to another. The cost could be evaluated purely upon the time or purely upon the distance; for more details see Dare [4].

### 4 Ant Colony Optimization for GPS Surveying Problem

Real ants foraging for food lay down quantities of pheromone (chemical cues) marking the path that they follow. An isolated ant moves essentially at random but an ant encountering a previously laid pheromone will detect it and decide to follow it with high probability and thereby reinforce it with a further quantity of pheromone. The repetition of the above mechanism represents the auto-catalytic behavior of real ant colony where the more the ants follow a trail, the more attractive that trail becomes (Figure 3).

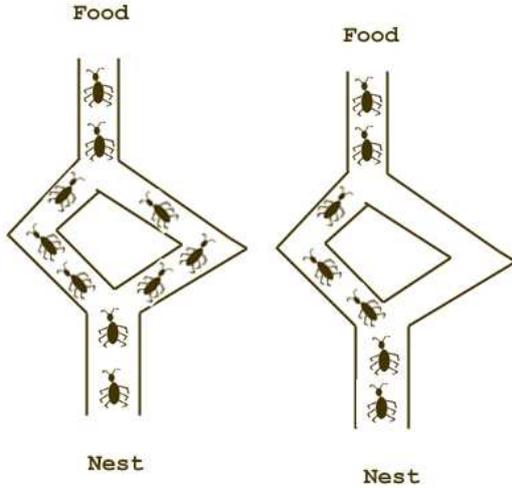


Figure 3: Real ants behavior

The ACO algorithm uses a colony of artificial ants that behave as cooperative agents in a mathematics space where they are allowed to search and reinforce pathways (solutions) in order to find the optimal ones. The problem is represented by graph and the ants walk on the graph to construct solutions. The solution is represented by path in the graph. After initialization of the pheromone trails, ants construct feasible solutions, starting from random nodes, then the pheromone trails are updated. At each step ants compute a set of feasible moves and select the best one (according to some probabilistic rules) to carry out the rest of the tour. The structure of ACO algorithm is shown in Figure 4. The transition probability  $p_{ij}$ , to choose the node  $j$  when the current node is  $i$ , is based on the heuristic information  $\eta_{ij}$  and pheromone trail level  $\tau_{ij}$  of the move, where  $i, j = 1, \dots, n$ .

$$p_{ij} = \frac{\tau_{ij}^{\alpha} \eta_{ij}^{\beta}}{\sum_{k \in U_{\text{unused}}} \tau_{ik}^{\alpha} \eta_{ik}^{\beta}} \quad (1)$$

The higher value of the pheromone and the heuristic information, the more profitable is to select this move and resume the search. In the beginning, the initial pheromone level is set to a small positive constant value  $\tau_0$  and then ants update this value after completing the construction stage. ACO algorithms adopt different criteria to update the pheromone level.

In our implementation we use MAX-MIN Ant System (MMAS) [12], and Ant Colony System (ACS) [5], which are ones of the best ant approaches. In MMAS the main is using fixed upper bound  $\tau_{max}$  and

## Ant Colony Optimization

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Initialize number of ants;
Initialize the ACO parameters;
while not end-condition do
    for k=0 to number of ants
        ant k starts from random node;
        while solution is not constructed do
            ant k selects a node with probability;
        end while
    end for
    Local search procedure;
    Update-pheromone-trails;
end while

```

Figure 4: Pseudocode for ACO

lower bound  $\tau_{min}$  of the pheromone trails. Thus accumulation of big amount of pheromone by part of the possible movements and repetition of same solutions is partially prevented. The main features of MMAS are:

- Strong exploration to the space search of the best found solution. This can be achieved by only allowing one single ant to add pheromone after each iteration, the best one.
- Wide exploration of the best solution. After the first iteration the pheromone trails are reinitialized to  $\tau_{max}$ . In the next iteration only the movements that belong to the best solution receive a pheromone, while other pheromone values are only evaporated.

The aim of using only one solution is to make solution elements, which frequently occur in the best found solutions, get large reinforcement. The pheromone trail update rule is given by:

$$\tau_{ij} \leftarrow \rho \tau_{ij} + \Delta \tau_{ij}, \quad (2)$$

$$\Delta \tau_{ij} = \begin{cases} 1/C(V_{best}) & \text{if } (i, j) \in \text{best solution} \\ 0 & \text{otherwise} \end{cases},$$

Where  $V_{best}$  is the iteration best solution and  $i, j = 1, \dots, n$ . To avoid stagnation of the search, the range of possible pheromone value on each movement is limited to an interval  $[\tau_{min}, \tau_{max}]$ .  $\tau_{max}$  is an asymptotic maximum of  $\tau_{ij}$  and  $\tau_{max} = 1/(1 - \rho)C(V^*)$ , while  $\tau_{min} = 0.087\tau_{max}$ . Where  $V^*$  is the

optimal solution, but it is unknown, therefore we use  $V_{best}$  instead of  $V^*$ .

In ACS, the pheromone corresponding to constructed by ants solutions first is decreased by the rule:

$$\tau_{ij} \leftarrow \rho\tau_{ij} + (1 - \rho)\tau_0 \quad (3)$$

Where  $i, j = 1, \dots, n$ . After that the pheromone corresponding to the best found solution is increased by the similar to the MMAS solution way. All other pheromone stay unchanged. The main features of ACS are:

- Strong exploration of the space search of the best found solution.
- The pheromone of the worse solutions is decreased and partially prevents their repetition.

In both implementations we use heuristic information equals to one over the cost of the session.

## 5 Experimental Results

In this section we analyze the experimental results obtained using MMAS algorithm described in previous section. Like a test problems we use real data from Malta and Seychelles GPS networks. The Malta GPS network is composed of 38 sessions and the Seychelles GPS network is composed of 71 sessions. We use 6 larger test problems too, from <http://www.informatik.uni-heidelberg.de/groups/comopt/software/TSLIB95/ATSP.html>. These test problems range from 100 to 443 sessions.

For every experiment, the results are obtained by performing 30 independent runs, then averaging the fitness values obtained in order to ensure statistical confidence of the observed difference. Analysis of the data using ANOVA/Kruskal-Wallis test has been used to get statistical confidence of the level 95% of the results.

In Table 1 we show the achieved costs for every test problem. For comparison reason we use same parameter settings for the both algorithms, as follows:  $\tau_0 = 0.005$ ,  $\alpha = 1$ ,  $\beta = 2$ ,  $\rho = 0.2$ , number of iterations is equal to the number of sessions, number of used ants is 10. With bold are the minimal achieved average costs.

Analyzing the results we conclude that MMAS algorithm outperforms ACS algorithm. Minimal costs are obtained by MMAS for the most of the tests. For the Malta and ftv170 there are not relative difference between the two algorithms.

Table 1: MMAS and ACS comparison

Tests	sessions	ACS	MMAS
Malta	38	924	923
Seychelles	71	920	<b>912</b>
rro124	100	41584	<b>41454</b>
ftv170	170	3395	3417
rgb323	323	1688	<b>1665</b>
rgb358	358	1734	<b>1675</b>
rgb403	403	3499	<b>3443</b>
rgb443	443	3806	<b>3754</b>

## 6 Conclusion

The GPS surveying problem is addressed in this paper. Instances containing from 38 to 443 sessions have been solved using MMAS and ACS algorithms. For both algorithms we use same parameter settings. A comparison of the performance of the both ACO algorithms applied on various GPS networks is reported. The MMAS algorithm outperforms ACS algorithm. The obtained results are encouraging and the ability of the developed techniques to generate rapidly high-quality solutions for observing GPS networks can be seen. The problem is important because it arises in wireless communications like GPS and mobile phone and can improve the services in the networks. Thus the problem has an economical importance.

In future work, other metaheuristics algorithms will be developed and applied and compared.

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