Intuitionistic Fuzzy Estimations of the Ant Colony Optimization

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1 Introduction

Combinatorial optimization is a branch of optimization. Its domain is optimization problems which set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution. A combinatorial optimization problem consists of objective function, which needs to be minimized or maximized, and constraints. Examples of optimization problems are Traveling Salesman Problem [6], Vehicle Routing [7], Minimum Spanning Tree [5], Constrain Satisfaction [4], Knapsack Problem [3], etc. They are NP-hard problems and in order to obtain solution close to the optimality in reasonable time, metaheuristic methods are used. One of them is Ant Colony Optimization (ACO) [2].

Real ants foraging for food lay down quantities of pheromone (chemical cues) marking the path that they follow. An isolated ant moves essentially at random but an ant encountering a previously laid pheromone will detect it and decide to follow it with high probability and thereby reinforce it with a further quantity of pheromone. The repetition of the above mechanism represents the auto-catalytic behavior of a real ant colony where the more the ants follow a trail, the more attractive that trail becomes. The ACO algorithm uses a colony of artificial ants that behave as cooperative agents in a mathematical space where they are allowed to search and reinforce pathways (solutions) in order to find the optimal ones. The problem is represented by graph and the ants walk on the graph to construct solutions. The solutions are represented by paths in the graph. After the initialization of the pheromone trails, the ants construct feasible solutions, starting from random nodes, and then the pheromone trails are updated. At each step the ants compute a set of feasible moves and select the best one (according to some probabilistic rules) to continue the rest of the tour. The structure of the ACO algorithm is shown by the pseudocode below. The transition probability $p_{i,j}$, to choose the node j when the current node is i, is based on the heuristic information $\eta_{i,j}$ and the pheromone trail level $\tau_{i,j}$ of the move, where $i, j =$

$1, \ldots, n$.

$$p_{i,j} = \frac{\tau_{i,j}^a \eta_{i,j}^b}{\sum_{k \in Unused} \tau_{i,k}^a \eta_{i,k}^b},$$

where $Unused$ is the set of unused nodes of the graph.
The higher the value of the pheromone and the heuristic information, the more profitable it is to select this move and resume the search. In the beginning, the initial pheromone level is set to a small positive constant value τ_0; later, the ants update this value after completing the construction stage. ACO algorithms adopt different criteria to update the pheromone level.

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Ant Colony Optimization
Initialize number of ants;
Initialize the ACO parameters;
while not end-condition do
    for k=0 to number of ants
        ant k choses start node;
        while solution is not constructed do
            ant k selects higher probability node;
        end while
    end for
    Update-pheromone-trails;
end while
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Figure 1: Pseudocode for ACO

The pheromone trail update rule is given by:

$$\tau_{i,j} \leftarrow \rho \tau_{i,j} + \Delta \tau_{i,j},$$

where ρ models evaporation in the nature and $\Delta \tau_{i,j}$ is new added pheromone which is proportional to the quality of the solution.
Our novelty is to use Intuitionistic Fuzzy Estimations (IFE see [1]) of start nodes with respect to the quality of the solution and thus to better menage the search process. We offer various start strategies and their combinations.

2 Start Strategies

The known ACO algorithms create a solution starting from random node. But for some problems, especially subset problems, it is important from which node the search process starts. For example if an ant starts from node which does not belong to the optimal solution, probability to construct it is zero. Therefore we offer several start strategies.
Let the graph of the problem has m nodes. We divide the set of nodes on N subsets. There are different ways for dividing. Normally, the graph are randomly enumerated.

An example for creating of the subset, without lost of generality, is: the node number one is in the first subset, the node number two in the second subset, etc. the node number N is in the $N-th$ subset, the node number $N+1$ is in the first subset, etc. Thus the number of the nodes in the separate subsets are almost equal. We introduce estimations $D_j(i)$ and $E_j(i)$ of the node subsets, where $i \geq 2$ is the number of the current iteration and $D_j(i)$ and $E_j(i)$ are weight coefficients of $j-th$ node subset ($1 \leq j \leq N$), which we calculate by the following formulas:

$$D_j(i) = \frac{i.D_j(i-1) + F_j(i)}{i},$$
$$E_j(i) = \frac{i.E_j(i-1) + G_j(i)}{i},$$

where $i \geq 1$ is the current process iteration and for each j ($1 \leq j \leq N$):

$$F_j(i) = \begin{cases} \frac{f_{j,A}}{n_j} & \text{if } n_j \neq 0 \\ F_j(i-1) & \text{otherwise} \end{cases}, \quad (1)$$

$$G_j(i) = \begin{cases} \frac{g_{j,B}}{n_j} & \text{if } n_j \neq 0 \\ G_j(i-1) & \text{otherwise} \end{cases}, \quad (2)$$

and $f_{j,A}$ is the number of the solutions among the best $A\%$, and $g_{j,B}$ is the number of the solutions among the worst $B\%$, where $A + B \leq 100$, $i \geq 1$ and

$$\sum_{j=1}^{N} n_j = n,$$

where n_j ($1 \leq j \leq N$) is the number of solutions obtained by ants starting from nodes subset j. Initial values of the weight coefficients are: $D_j(1) = 1$ and $E_j(1) = 0$. Obviously, $F_j(i)$, $G_j(i)$, $F_j(i)$ and $G_j(i) \in [0, 1]$, i.e., they are IFEs.
We try to use the experience of the ants from previous iteration to choose the better starting node. Other authors use this experience only by the pheromone, when the ants construct the solutions. Let us fix threshold E for $E_j(i)$ and D for $D_j(i)$, than we construct several strategies to choose start nod for every ant, the threshold E increase every iteration with $1/i$ where i is the number of the current iteration:

1 If $E_j(i) > E$ then the subset j is forbidden for current iteration and we choose the starting node randomly from $\{j \mid j$ is not forbidden$\}$;

2 If $E_j(i) > E$ then the subset j is forbidden for current simulation and we choose the starting node randomly from $\{j \mid j$ is not forbidden$\}$;

3 If $E_j(i) > E+$ then the subset j is forbidden for K_1 consecutive iterations and we choose the starting node randomly from $\{j \mid j$ is not forbidden$\}$;

4 If $E \geq E_j(i)$ and $D \geq D_j(i)$ for K_2 consecutive iterations, then the subset j is forbidden for current simulation and we choose the starting node randomly from $\{j \mid j$ is not forbidden$\}$;

5 Let $r_1 \in [0.5, 1)$ is a random number. Let $r_2 \in [0, 1]$ is a random number. If $r_2 > r_1$ we randomly choose node from subset $\{j\ |D_j(i) > D\}$, otherwise we randomly chose a node from the not forbidden subsets, r_1 is chosen and fixed at the beginning.

6 Let $r_1 \in [0.5, 1)$ is a random number. Let $r_2 \in [0, 1]$ is a random number. If $r_2 > r_1$ we randomly choose node from subset $\{j\ |D_j(i) > D\}$, otherwise we randomly chose a node from the not forbidden subsets, r_1 is chosen at the beginning and increase with r_3 every iteration.

Where $0 \leq K_1 \leq$"number of iterations" is a parameter. If $K_1 = 0$, than strategy 3 is equal to the random choose of the start node. If $K_1 = 1$, than strategy 3 is equal to the strategy 1. If $K_1 =$"maximal number of iterations", than strategy 3 is equal to the strategy 2.
We can use more than one strategy for choosing the start node, but there are strategies which can not be combined. We distribute the strategies into three sets: $St1 = \{strategy1,\ strategy2,\ strategy3\}$, $St2 = \{strategy4\}$ and $St3 = \{strategy5,\ strategy6\}$. The strategies from same set can not be used at once. Thus we can use strategy from one set or combine it with strategies from other sets. Exemplary combinations are $(strategy1)$, $(strategy2;\ strategy5)$, $(strategy3;\ strategy4;\ strategy6)$.
In this paper we address the modelling of the process of ant colony optimization method by using fuzzy estimations, combining six start strategies. So, the start node of each ant depends of the goodness of the respective region. In a future we will focus on parameter settings which manage the starting procedure. We will investigate on influence of the parameters to algorithm performance.

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