BULGARIAN ACADEMY OF SCIENCES
INSTITUTE OF INFORMATION AND COMMUNICATION TECHNOLOGIES
Acad. G. Bonchev Str., block 2, 1113 Sofia, BULGARIA
Phone: +359 2 979 66 11, +359 2 870 84 94; Fax: +359 2 870 72 73
E-mail: iict@bas.bg

SEMINAR

Professor Joseph Pasciak, Texas A&M University, TX, US, will give a lecture in the seminar room 218 on 17 August, 14:00.

Rational approximations to functions involving fractional powers of elliptic operators.

Abstract: We start with an unbounded elliptic operator L with a homogeneous boundary condition, e.g., the negative Laplacian on a bounded domain Ω in R^d with homogeneous Dirichlet boundary conditions. Our goal is to numerically approximate functions of L applied to data $f \in L^2(\Omega)$, for example, $u = L^{-\alpha} f$, $u = Exp(-tL^\alpha)$, or $u = e_{\alpha,1}(-t^\alpha L^\beta) f$ with $e_{\alpha,\gamma}$ denoting the Mittag-Leffler function. We apply the finite element method to obtain a discrete approximation to L denoted by L_h with h being the finite element mesh size. Our semidiscrete approximation is u_h which is defined by replacing L with L_h and f with $P_h f$ with P_h denoting the $L^2(\Omega)$ projection into the finite element subspace. The rational approximations that we consider give rise to norm convergence, e.g., $\|u_h - R_n(P_h f)\|_{L^2(\Omega)} \le \epsilon(n)\,\|f\|_{L^2(\Omega)}$ with n denoting the degree of the polynomials in the rational approximation $R_n(\cdot)$. Such estimates follow from estimates on the scalar problem, e.g., $|\lambda^{-\alpha} - R_n(\lambda)| \le \epsilon(n)$ holding uniformly for $\lambda \ge \lambda_0$ where λ_0 denotes the smallest eigenvalue of L_h.

The first set of rational approximations are defined by applying sinc quadrature to Dunford-Taylor like expansions of u_h. Analysis given by Bonito-Lei-Pasciak (BLP) shows that such schemes converge exponentially in the number of integration points, i.e., the degree of the polynomials in the rational approximation. The second scheme is related to a method proposed by Vabaschevich which reformulates the first case problem $u_h = L_h^{-\alpha} P_h f$ as the solution of a parabolic problem and subsequently applies a time stepping method to the resulting time dependent equation. This leads to a second class of rational approximations again with exponential convergence in n as shown in Duan-Lazarov-Pasciak but requires rational polynomials of degree $O(n\log(h^{-1}))$. The final scheme is based on replacing L_h by $T_h := L_h^{-1}$ so that $u_h = T_h^\alpha P_h f$, $u_h = Exp(-tT_h^{-\alpha})P_h f$ and $u_h = e_{\alpha,1}(-t^\alpha T_h^{-\beta})P_h f$, respectively. We consider defining R_n to be the best uniform rational approximation to x^α, $Exp(-t/x)$ and $e_{\alpha,1}(-t^\alpha x^{-\beta})$, respectively, on the interval $[0, \lambda_0^{-1}]$ for the first two and $[0, \lambda_0^{-\beta}]$ for the last. The best uniform approximation property implies that these methods cannot converge slower than the exponential convergence of the first case above. We conclude the talk with numerical results illustrating the theoretical convergence.