Numerical Analysis Of Multilevel Monte Carlo For Scalar Jump-diffusion SDEs

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The multilevel Monte Carlo method for finite intensity rate jump-diffusion SDEs has been proposed for estimating the value of financial options and its efficiency demonstrated numerically in previous work. In this paper we analyse the conditions required by the theorem on the computational complexity of the multilevel method using the jump-adapted Milstein discretisation, to verify the weak convergence of the estimator and to determine or bound the order of convergence of the variance of the multilevel correction estimator.

Giles, Debrabant and Rößler have derived variance convergence rates for a variety of financial payoffs arising from the solution of scalar SDEs using MLMC based on the Milstein discretisation. Numerical results suggest that all of these results are near-optimal, and they lead to an RMS error of ϵ being obtained with a computational complexity of $O(\epsilon^{-2})$.

Xia extended the MLMC approach to finite rate jump-diffusion SDEs using a jump-adapted Milstein discretisation scheme for the constant rate case and a thinning procedure for the bounded state-dependent intensity case. The numerical results indicate a computational complexity of $O(\epsilon^{-2})$. This paper supports this through numerical analysis to bound the convergence of the multilevel correction variance. In more detail, we bound the variance for European call, Asian, lookback, barrier and digital options in the constant jump rate case. We also extend the analysis to cover the convergence for the case of bounded state-dependent intensities. Consequently, the same $O(\epsilon^{-2})$ computational complexity is proved for all cases except for the barrier option for which the best that can currently be proved is a near first order convergence of the variance, leading to the computational complexity being $o(\epsilon^{-2-\delta})$ for any strictly positive δ.