Monte Carlo Methods Based on Analytic Extension of the Resolvent of the Helmholtz Equation

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The article presents the results obtained by applying the analytic extension of the Monte Carlo methods for solving the Helmholtz equation by replacing the spectral parameter.

The standard “walk on spheres” estimates for solving the Helmholtz equation are created by the reduction of the original problem to a Fredholm integral equation of the second kind. The substantiation of estimates for linear functionals of solutions based on the convergence corresponding Neumann series, which leads to additional restrictions on the input parameters of the problem at whose spectral radius of the corresponding integral operator is obliged to be less than unity. These estimates are not satisfactory because of insufficient rate of convergence of the Neumann series, when the spectral radius is fairly close to unity.

In this paper, a new “walk-on spheres” estimates and probabilistic representation for solutions of the Helmholtz equation was constructed by analytical continuation of the resolvent in case of divergence of classical methods.

The novelty of the work consists in the application the G.A. Mikhailov approach of parametric differentiation to build a scalar “walk on spheres” estimates. It is shown that the necessary estimates of iterations of the resolvent can be obtained by parametric differentiation of the special boundary value problem. On this approach the analytic extension of estimates for solutions and the covariance function of metaharmonic equations with random right-hand side and interior Dirichlet boundary condition were constructed.

The optimal parameters (number of trajectories, value determining the boundary error, the number of iterations) were obtained. The numerical results confirming the theoretical assertions are presented.