Monte Carlo Algorithms for Parameter Dependent Integration Problems

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We study the problem of indefinite integration of functions depending on a parameter. By this we mean the following: We are given a function $f \in C^r([0,1]^{d_1+d_2})$ and we want to compute (approximately)

$$\int_{[0,x]} f(s,t)\,dt \qquad (s \in [0,1]^{d_1}, x \in [0,1]^{d_2}),$$

simultaneously for all $x \in [0,1]^{d_2}$ and for all $s \in [0,1]^{d_1}$, i.e., the indefinite integral (or anti-derivative) with respect to the second variable for all parameter values s at once. We want our approximation to hold uniformly for all $x \in [0,1]^{d_2}$ and all parameter values $s \in [0,1]^{d_1}$.

We propose and analyze an algorithm based on a combination of Smolyak interpolation with simultaneous Monte Carlo sampling and variance reduction.

In the framework of information based complexity the problem of definite parametric integration (i.e., $x = 1$) was previously studied by S. Heinrich and E. Sindambiwe. They presented a randomized algorithm for the problem with optimal order (only in one single case a logarithmic gap remains). Moreover, recently S. Heinrich and B. Milla showed that the optimal order for indefinite integration without dependence on a parameter is $n^{-1/2}$, the same as for definite integration.

We now improved our previous result and provide a randomized algorithm for the general problem defined above having optimal order with (almost) exact logarithmic factors. In other words, we obtain the same optimal order as for definite parametric integration and only in one single case (i.e., $r = \frac{d_1}{2}$) a gap of a logarithmic factor remains.