Numerical analysis of multilevel Monte Carlo Greeks

Sylvestre Burgos, Michael B. Giles (University of Oxford, U.K.)

May 2011

Earlier numerical evidence suggests that multilevel Monte Carlo path simulation offers an efficient way to reduce the complexity of the calculation of Greeks. We propose a supporting analysis of multilevel Monte Carlo's complexity in this context.

The pathwise sensitivity analysis differentiates the path evolution and reduces the smoothness of the payoff by one order. Computing a first order Greek for a call option is similar to pricing a digital option whose discontinuity induces an increased computational cost; a naive implementation for digital options' Greeks is impossible because of the inapplicability of pathwise sensitivity to discontinuous payoffs.

This can be addressed in different ways: payoff smoothing using conditional expectations of the payoff before maturity; an approximation of the above technique using path splitting for the final timestep; the use of a hybrid combination of pathwise sensitivity and the likelihood ratio method.

The computation of Greeks for exotic payoffs also presents new challenges. As before, naive use of multilevel algorithms does not yield maximum computational savings. We present ways in which Brownian interpolation can be used in combination with multilevel methods to get efficient estimators of the Greeks for barrier and lookback options.