Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

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August 29 - September 2, 2011

Monte Carlo (MC) simulation employing Latin Hypercube Sampling (LHS) is one of the most popular modelling tools. While its application in areas like experimental design is well justified, the efficiency of LHS in other areas such as high dimensional integration can be no better than the standard MC method based on random numbers. To provide a high efficiency of high dimensional integration, high uniformity of sampled points is required. LHS being well stratified in one dimension is not designed to provide good uniformity properties in high dimensions. It is known that for high dimensional integrals the convergence rate of the MC estimates based on random sampling is $O(N^{-\frac{1}{2}})$, where N is the number of sampled points. A higher rate of convergence can be obtained by using Quasi Monte Carlo (QMC) methods based on low-discrepancy sequences. Asymptotically, QMC can provide the rate of convergence $O(N^{-1})$. In this work we compare efficiencies of three sampling methods: the MC method with random and LHS sampling and the QMC method with sampling based on Sobol' sequences. We apply the high-dimensional Sobol' sequence generator with advanced uniformity properties: Property A for all dimensions and Property A' for adjacent dimensions. It was developed jointly with I. Sobol'. Firstly we compare L_2 discrepancies and show that the QMC method has the lowest discrepancy up to dimension 20. Secondly, we use a number of test functions of various complexities for high dimensional integration. Using global sensitivity analysis, functions are classified with respect to their dependence on the input variables: functions with not equally important variables (type A), functions with equally important variables and with dominant low order terms (type B) and functions with equally important variables and with dominant interaction terms (type C). Comparison shows that for types A and B functions, convergence of the QMC method is close to $O(N^{-1})$ while the MC method with random and LHS sampling has the convergence rate close to $O(N^{-\frac{1}{2}})$. For types C functions, the convergence rate of the QMC method drops. However the QMC method still remains the most efficient method among the three sampling techniques.