MULTI-LEVEL MONTE CARLO SIMULATIONS OF MEAN EXIT TIMES

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Abstract. Many financial quantities such as path dependent options, volatility swaps and credit risk models are based on simulations of stopping times. One has to use Monte Carlo simulations especially when a high number of dimensions is involved. However, numerical methods for stochastic differential equations are relatively inefficient when used to approximate mean exit times. In particular, although the basic Euler–Maruyama method has weak order equal to one for approximating the expected value of the solution, the order reduces to one half when it is used in a straightforward manner to approximate the mean value of a (stopped) exit time. Consequently, the widely used standard approach of combining an Euler-Maruyama discretization with a Monte Carlo simulation leads to a computationally expensive procedure. In this work, we show that the multi-level approach developed by Giles (Operations Research, 2008) can be adapted to the mean exit time context. In order to justify the algorithm, we analyse the strong error of the discretization method in terms of its ability to approximate the exit time. We then show that the resulting multi-level algorithm improves the expected computational complexity by an order of magnitude, in terms of the required accuracy. Computational results are provided to illustrate the analysis.

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