Multilevel Monte Carlo For Lévy Driven SDEs

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We consider the computational problem of evaluating an expectation $\mathrm{E}[f(Y)]$, where $Y = (Y_t)_{t\in[0,1]}$ is the solution of a stochastic differential equation driven by a square integrable Lévy process and $f : D[0,1] \to \mathbb{R}$. Here, $D[0,1]$ denotes the Skorohod space of càdlàg functions. Computational problems of this kind arise, e.g., in computational finance for valuations of path-dependent options.

We introduce multilevel Monte Carlo algorithms for the Euler scheme where the small jumps of the Lévy process are neglected. Upper bounds for the worst case error of these multilevel algorithms are provided in terms of their computational cost. Here, the worst case is taken over all functionals f, that are Lipschitz continuous with Lipschitz constant one with respect to supremum norm. Specifically, if the driving Lévy process has Blumenthal-Getoor index β, errors of order $n^{-(\frac{1}{\beta\vee 1}-\frac{1}{2})}$ are achieved with cost n. In the case $\beta > 1$, this rate can even be improved by using a Gaussian correction term for the neglected small jumps.

In this talk, we emphasize on the implementation of the multilevel algorithm and we present numerical experiments complementing the theoretical results. Hereby, suitable bias and variance estimation allows to adaptively choose the replication numbers for the different levels.