Sensitivity Analysis of Compact Models in Nanodevice Modeling

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Introduction

Motivation

Statistical variability in transistors

Sensitivity analysis technique

Numerical experiments

Open problems
Motivation

Simulation models are used (in diagnostic or prognostic fashion) in many fields to understand complex phenomena (natural or social) and consequently as tools to support decisions and policy.

Our knowledge is often flawed by uncertainties (partly irreducible, largely unquantifiable), imperfect understanding, subjective values.

We need tools to scrutinize uncertainties in model inputs, assumptions, models structures, to see how they propagate and affect inferences (that are used for policy decisions).
Compact model parameters (in circuit simulations)

- Compact model parameter generation
- Statistical compact model parameter extraction
Definition of sensitivity analysis (SA)

"The study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input."

- Metamodeling
- Sensitivity analysis techniques
  - Local approach (one-at-a-time experiments)
  - Screening methods
  - Variance-based methods - Sobol’ approach, FAST
  - Derivative-based global sensitivity measures
- Monte Carlo / quasi-Monte Carlo methods
  - Plain Monte Carlo Algorithm
  - Adaptive Monte Carlo Algorithm
  - Sobol’ Quasi Monte Carlo Algorithm
  - Monte Carlo Algorithm Based on Sobol’ Sequences
- Pseudo / quasi-random number generators
The mathematical model

\[ u = f(x), \quad \text{where} \quad x = (x_1, x_2, \ldots, x_d) \in U^d \equiv [0, 1]^d \]

is a vector of inputs with a joint p.d.f. \( p(x) = p(x_1, \ldots, x_d) \).
The mathematical model

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Total Sensitivity Index of input parameter \( x_i, \quad i \in \{1, \ldots, d\} \):

\[ S_{x_i}^{\text{tot}} = S_i + \sum_{l_1 \neq i} S_{il_1} + \sum_{l_1, l_2 \neq i, l_1 < l_2} S_{il_1l_2} + \ldots + S_{il_1\ldots l_{d-1}}, \]

where

\( S_i \) - the main effect (first-order sensitivity index) of \( x_i \) and
\( S_{il_1\ldots l_{j-1}} \) - \( j^{\text{th}} \) order sensitivity index for parameter \( x_i \) (\( 2 \leq j \leq d \)).
ANalysis Of VAriances (ANOVA) HDMR of a square integrable function $f(x)$:

$$f(x) = f_0 + \sum_{\nu=1}^d \sum_{l_1<\ldots<l_{\nu}} f_{l_1\ldots l_{\nu}}(x_{l_1}, x_{l_2}, \ldots, x_{l_{\nu}}),$$

where $f_0 = \text{const}$,

and

$$\int_0^1 f_{l_1\ldots l_{\nu}}(x_{l_1}, x_{l_2}, \ldots, x_{l_{\nu}})dx_{l_k} = 0, \quad 1 \leq k \leq \nu, \quad \nu = 1, \ldots, d.$$

The functions in the right-hand side are defined in a unique way:

- $f_0 = \int_{U^d} f(x)dx, \quad f_1(x_{l_1}) = \int_{U^{d-1}} f(x) \prod_{k \neq l_1} dx_k - f_0, \quad l_1 \in \{1, \ldots, d\}$

- $\int_{U^d} f_{i_1\ldots i_\mu} f_{j_1\ldots j_\nu} dx = 0, \quad (i_1, \ldots, i_\mu) \neq (j_1, \ldots, j_\nu), \quad \mu, \nu \in \{1, \ldots, d\}$. 
Definition (Sobol')

\[ S_{l_1 \ldots l_\nu} = \frac{D_{l_1 \ldots l_\nu}}{D}, \quad \nu \in \{1, \ldots, d\}, \]

where

- partial variances
  \[ D_{l_1 \ldots l_\nu} = \int f_{l_1 \ldots l_\nu}^2 \, dx_{l_1} \ldots dx_{l_\nu}, \]

- total variance
  \[ D = \int_{U^d} f^2(x) \, dx - f_0^2, \quad D = \sum_{\nu=1}^d \sum_{l_1 < \ldots < l_\nu} D_{l_1 \ldots l_\nu}, \]

and the following properties hold:

- \[ S_{l_1 \ldots l_s} \geq 0, \quad \sum_{s=1}^d \sum_{l_1 < \ldots < l_s} S_{l_1 \ldots l_s} = 1. \]
BSIM4 compact model

Statistical variability

Sensitivity analysis of a submodel

- $V_{th0}$ - basic long-channel threshold voltage parameter
- $U_0$ - low-field mobility parameter
- $R_{ds}$ - basic source/drain resistance parameter
- $D_{sub}$ - drain-induced barrier-lowering (DIBL) parameter
Random number generation - SIMLAB v2.2
Produce a database with model outputs
Computing Sobol’ sensitivity indices - SIMLAB v2.2

SIMLAB: http://simlab.jrc.ec.europa.eu/
# Sobol indices

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<td>0.359418</td>
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<td>0.047362</td>
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<tr>
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**Figure:** General procedure for sensitivity analysis.

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Sobol indices

- $y$: Closed
- $y$: Total

1. $x_1$: 0.429220 (0.429220)
2. $x_2$: 0.066087 (0.066087)
3. $x_3$: 0.022617 (0.022617)
4. $x_1,x_2$: 0.245330 (0.740638)
5. $x_1,x_3$: 0.157343 (0.609180)
6. $x_2,x_3$: 0.023386 (0.112090)
7. $x_1,x_2,x_3$: 0.056016 (1.000000)

Total:
- $x_1$: 0.892491
- $x_2$: 0.386237
- $x_3$: 0.258302
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<table>
<thead>
<tr>
<th>Sample size</th>
<th>x1 (Vth0)</th>
<th>x2 (U0)</th>
<th>x3 (Dsub)</th>
<th>Total x1</th>
<th>Total x2</th>
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Figure: General procedure for sensitivity analysis.
Uniform or normal distribution of inputs?
Approximation of model database
Orthogonality (independence) of inputs