Advanced Statistical Strategy for Generation of Non-Normally distributed PSP Compact Model Parameters and Statistical Circuit Simulation


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** Gold Standard Simulations (GSS) Ltd.
Summary

- Background
- Physical simulation
- Compact model extraction
- Principle Component Analysis
- Nonlinear Power Method
- Conclusions
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CMOS variability classification

After K. Takeuchi (NEC)
Sources of statistical variability

\[ I_D = (V_S, V_D, V_G) \]
Sources of statistical variability

Random dopants  Polysilicon/high-k Granularity  Line edge roughness
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GSS ‘atomistic’ simulation tools

- **3D DD simulator**
  - Random discrete dopants
  - Random interface roughness
  - Line edge roughness
  - DG quantum corrections

- **3D MC simulator**
  - Si/S-Si/SiGe/III-V
  - New interface scattering models
  - Degeneracy
  - High-\textit{k} dielectrics
  - \textit{Ab-initio} impurity scattering
  - \textit{Ab-initio} interface roughness

- **3D NEGF simulator**
  - Full 3D NEGF
  - Coupled mode space 3D NEGF
  - Includes scattering
The basic semiconductor equations

The basic equations that describe the operation of most semiconductor devices are:

\[
\begin{align*}
\frac{d^2 \psi}{dx^2} &= -\frac{q}{\varepsilon_{Si}} \left[ p(x) - n(x) + N_D^+ - N_A^- \right] \quad \text{Poisson's equation} \\
\frac{dn}{dt} &= \frac{1}{q} \frac{\partial J_n}{\partial x} - R_n + G_n \\
\frac{dp}{dt} &= -\frac{1}{q} \frac{\partial J_p}{\partial x} - R_p + G_p
\end{align*}
\]

The continuity equations for electrons and holes based on conservation of mobile charge.

Where

\[
\begin{align*}
J_n &= -qn \mu_n \left( \frac{d\psi}{dx} - \frac{k_B T}{qn} \frac{dn}{dx} \right) = -qn \mu_n \frac{d\phi_n}{dx} \\
J_p &= -qp \mu_p \left( \frac{d\psi}{dx} + \frac{k_B T}{qp} \frac{dp}{dx} \right) = -qp \mu_p \frac{d\phi_p}{dx}
\end{align*}
\]

\[
\begin{align*}
\phi_n, \phi_p & \quad \text{quasi-Fermi potentials} \\
\phi_n &= \psi - \frac{k_B T}{q} \ln \left( \frac{n}{n_i} \right) \\
\phi_p &= \psi + \frac{k_B T}{q} \ln \left( \frac{p}{n_i} \right)
\end{align*}
\]
Grid/cluster based simulation technology

Automated job submission

Job monitoring

Successful completion

Data harvesting in a database

Statistical analysis

Visualisation

Computational steering

Yes

No

2380 CPUs
Statistical simulation results
35 nm MOSFET

**RDD+LER+PSG**
Compact models

\[ V_D = 50 \text{mV} \]

\[ V_D = 1.0 \text{V} \]
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Compact models

- Compact models (CM) used in circuit simulators like SPICE are the interface between technology and design.

- CM are usually closed form analytical expressions returning terminal currents as a function of applied bias.

- CM have a large number of parameters determined by fitting to measured or simulated transistor characteristics.

- The industrial standard compact models are BSIM and PSP.

\[ I_D = (V_S, V_D, V_G, p1, p2, \ldots pn) \]
Two stage parameter extraction

Large set of microscopically different transistors
Comprehensive sensitivity analysis
Statistical accuracy

BSIM

PSP

Error (%) vs Frequency for different parameter statistical extraction methods.
### Statistical accuracy

<table>
<thead>
<tr>
<th>PSP</th>
<th>BSIM</th>
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<tbody>
<tr>
<td>Vth</td>
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<tr>
<td>Ion</td>
<td></td>
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<tr>
<td>Ioff</td>
<td></td>
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<tr>
<td>DIBL</td>
<td></td>
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<td>SS</td>
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### Statistical compact model parameter correlations

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<th>Vth0</th>
<th>U0</th>
<th>Voff</th>
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<tr>
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<tr>
<td>Nfactor</td>
<td>Minv</td>
<td>Rdsw</td>
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</table>
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PCA

PCA converts a set of observations of correlated variables into a set of values of uncorrelated variables called Principle components.

Center 1,3
First PC in (0.88,0.48) direction σ=3
Second PC in orthogonal direction σ=1
Naïve approach vs. PCA

Naïve 85%, PCA 15%

Naïve 25%, PCA 5%
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The compact model parameters are not normally distributed

\[ \gamma_1 = 1.61 \]
\[ \gamma_2 = 5.17 \]

\[ \gamma_1 = 1.28 \]
\[ \gamma_2 = 4.76 \]
The Nonlinear Power Method (NPM)

- The NPM preserves the correlations and reproduces the higher moments of the SCM parameter distributions
- The NPM generates multivariate non-normal distributions with an arbitrary covariance matrix from a set of analytical equations
The Nonlinear Power Method (NPM)

\[ Y_i = c_i^T Z_i \]

\[ E[Y_i] = c_i^T E[Z_i] \quad \text{Average} \]

\[ \text{VAR}[Y_i] = E\left[ (\frac{(c_i^T Z_i)^2}{E[c_i^T Z_i]^2}) \right] \quad \text{Variance} \]

\[ \gamma_{1i} = \frac{E\left[ (c_i^T Z_i)^3 \right] - 3E\left[ (c_i^T Z_i)^2 \right]E[c_i^T Z_i] + 2E[c_i^T Z_i]^3}{(\text{VAR}[Y_i])^{3/2}} \quad \text{Skew} \]

\[ \gamma_{2i} = \frac{E\left[ (c_i^T Z_i)^4 \right] - 4E\left[ (c_i^T Z_i)^3 \right]E[c_i^T Z_i] - 3E\left[ (c_i^T Z_i)^2 \right]^2 + 12E\left[ (c_i^T Z_i)^2 \right]E[c_i^T Z_i]^2 + 6E[c_i^T Z_i]^4}{(\text{VAR}[Y_i])^2} \quad \text{Kurtosis} \]
The nonlinear power method (NPM) can cope with non normal distributions.
NPM can cope also with the correlations

NPM

CTO

CSO

RDSW1

PCA
NPM reproducers the distribution of important figures of merit
Energy distribution of an inverter

![Energy distribution graph]

- **VSS**
- **VDD**
- **Vin**
- **Vout**
- **n**
- **p**

The graph shows the energy distribution for different methods:
- PCA
- 3 MOMENTS
- 4 MOMENTS
- EXTRACTED
Timing distribution in an inverter
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- The statistical compact model parameters are correlated.
- The distribution of the individual parameters deviate from normal.
- PCA fails to reproduce the proper distribution and correlation of the statistical compact model parameters.
- NPM not only accurately reproduces the accurately the parameters distribution and correlations but transistor figures of merit and circuit simulation results.