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Spline interpolation for modelling of accumulated effects of ozone

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Abstract: An alternative approach for computing exposure indices, such as AOT40 of the ground-level ozone, is considered in this paper. The aim of the study is to compare it with the traditional way of calculating the exposure indices in order to discover possible weaknesses or strengths of both approaches. The problem under consideration is of great importance when modelling results, obtained after running large-scale air pollution models, have to be compared with experimental data. Usually the data are received in a form of mesh-function. At the same time, accumulated effects have to be considered as integrals of smooth (at least continuous) functions. Cubic spline interpolation over data of ozone concentrations is considered as a tool for computing accumulated effects. Practical computations include estimations for measurements of several stations over Europe for two different years. The results obtained with the proposed procedure are compared with those achieved from the calculations according to the classical definition.

Keywords: environmental pollution; spline interpolation; ozone indexes; alternative AOT calculation; error analysis.

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1 Introduction

The reliability of large-scale mathematical models is an important issue when such models are used to help decision makers. At the same time some models use alternative ways to compute accumulated effects of ground-level ozone. Ground-level ozone is one of the most serious air pollutants in Europe today. High levels of ozone can affect the respiratory system and increase morbidity and mortality, particularly in sensitive groups of the population. Ozone also damages vegetation, reduces crop yields and corrodes building materials. Damage to agricultural crops caused by ozone is a well-documented problem in Southern Europe (Chervenkov, 2011) and can be catastrophic for farmers. Ozone damage can reduce both crop yield and quality, lowering the value of the crop. The effect of ozone on plants varies with many factors, including plant age, light levels, humidity and soil conditions, but studies indicate that the combination of peak ozone levels and the duration of exposure to high levels are the most significant factors. Exposure-based ozone metrics are still a most used practical measure for summarising ambient air quality.

The AOT40 for agricultural crops (AOT40c) is defined as a sum of the hourly ozone concentration exceeding the threshold of 40 ppb over the period from 1st May to 31st July [a recent revision of the definition considers the vegetative period, which varies according to the species and the geographical location United Nations Economic Commission for Europe (UN/ECE) (Directive 2002/3/EC, 2002)]. For the purposes of this paper, however, we maintained a larger period – 1st April–30th September, which is prescribed for forest protection (AOT40f).

It is more or less accepted that the ozone (O_3) index (or measure) AOT40 remains the basis for estimating the potential risk of forests or crops due to O_3, and for setting environmental quality objectives within the European Union (EU) and the UN/ECE (e.g., Directive 2002/3/EC, 2002; UN/ECE, 2004). One should mention some criticism of this statement (see, for instance Matyssek and Innes, 1999; Matyssek et al., 2004; Gerosa et al., 2007).

Environmental security is presently becoming a significant topic of interest all over the world. Both reliable models and physical experiments are now used to analyse carefully the most important physical and chemical processes during the transport and transformations of air pollutants. It is necessary to carry out many comprehensive scientific studies. Effective performance of such complicated procedures requires a joint research and collaboration between experts in the field of environmental modelling, numerical analysis and scientific computing.

Within this perspective, the knowledge about AOT40 values at forest monitoring sites is of considerable interest. Proper calculation of AOT40 implies the availability of complete hourly O_3 measurements throughout a 6-month period. Unfortunately, hourly O_3 measurement is uncommon in forests, a situation in which passive sampling offers considerable practical advantages (Krupa, 1989; Karnosky et al., 2007). Given the nature of the technique, passive sampling typically results in time-integrated data, such as weekly-to-monthly O_3 concentration, which are not consistent with the definition of AOT40 (see Ferretti and Gerosa, 2002; Zlatev and Dimov, 2006). For this reason, there were several attempts to estimate hourly values and cumulative or summation indices (as the AOT40) starting from integrated mean O_3 concentration obtained from passive samplers (Gerosa et al., 2001; Krupa and Nosal, 2002; Krupa et al., 2001, 2003).

The aim of the present work is to compare the alternative approach with the traditional way of calculating the exposure indices in order to discover possible weaknesses or strengths of both approaches. The problem under consideration is of great importance when modelling results, obtained after running large-scale air pollution models have to be compared to experimental data. Usually the data are received in a form of mesh-function. At the same time exposure indices, or accumulated effects, have to be considered as integrals of smooth (at least continuous) functions. Cubic spline interpolation over data of ozone concentrations is considered as a tool for computing accumulated effects AOT(c). The definition of AOT(c) can be considered as a quadrature formula for a real-life function, describing the ozone concentrations in time. *A priori* and *a posteriori* error estimates are considered and analysed. Practical computations include estimates of AOT(c) for measurements of several stations over Europe for two different years. The results obtained with the proposed procedure are compared with those achieved from the calculations according to the classical definition.

The rest of the paper is organised as follows. In Section 2 the problem under consideration is defined. Definitions of the traditional and alternative approaches of AOT(c) are given. It is shown that the traditional discrete formula can be considered as a first order numerical method for the computation of an approximation of an integral. In Section 3 numerical algorithm for computing coefficients of cubic spline interpolation over a mesh-function of hourly ozone concentrations is given. The numerical procedure used in this work is described. In Section 4 we present numerical results of our study. In Section 5 the applicability of the proposed approach is discussed. Some concluding remarks are given in Section 6.

2 Problem setting

As mentioned above, AOT40 is the accumulated amount of ozone over the threshold value of 40 ppb, i.e.,

$$AOT40 = \sum_{i=1}^{N} \max(c_i - 40, 0), \tag{1}$$

where N is the number of day-time hours [between 8:00 and 20:00 central European time (CET), according to Directive 2002/3/EC of the European Parliament (2012), which corresponds to 07:00-19:00 coordinated universal time (UTC)] in the period under consideration. The values c_i are calculated or measured ozone concentrations. The *max* symbol ensures that only ozone concentration values exceeding 40 ppb are included. The corresponding unit is $ppb \times hours$ (abbreviated to ppb.h). It is obvious that if one deals with experimental data, a mesh function defining the values of ozone concentrations at every hour should be used. Definition of the critical ozone levels for crops is based on Ozone Transport Commission (OTC, http://www.otcair.org/) results. It has, for practical reasons, been assumed that the critical exposure level for crops is 3,000 ppb.h, and losses of crops will be generally avoided if the AOT40 values do not exceed this level. Model calculations, however, show that for most European regions, AOT40 values are on the order of 15,000–25,000 ppb.h, and there is an enormous inter-annual variability. Therefofore, statistical variability will include a significant spatial and temporal characteristics. In addition, the selection of 40 ppb as the practically accepted threshold, rather than a small deviation from this value, such as 39 ppb or 41 ppb, for example, adds an additional level of uncertainty which must be accounted for in this analysis. One possibility to perform this is to apply well developed sensitivity analysis techniques (Saltelli et al., 2004, 2008; Saltelli, 2002; Sobol, 2001; Dimov, 2008; Dimov et al., 2010; Homma and Saltelli, 1996; Ostromsky et al., 2011).

We generalise the threshold value by considering the concentration as a variable, c, such that ozone-induced damage to forests or crops is a function of AOT(c). Note for reference that AOT40 values (1) are equivalent to AOT(c) when the parameter c = 40 ppb. The function $AOT(c)$ may be extended to a practical form, if it is defined by the sum:

$$AOT(c) = \sum_{i=1}^{N} \max(c_i - c, 0), \tag{2}$$

where N is the number of day-time hours in the period under consideration, c_i is the ozone concentration calculated by some model or measured at some station. If a mathematical model is used, then the AOT(c) values can be calculated for each grid-square of the model domain. The AOT(c) function given by (2) is based on the use of discrete values of the concentrations (hourly mean values). Often this function is defined using a continuous representation of the concentrations; see, for example, Simpson et al. (1997). The continuous representation given below is a slight generalisation of the commonly used definition [see again Simpson et al. (1997) for the commonly used definition]:

$$AOT(c(t)) = \sum_{i=1}^{M} \left\{ \int_{t_i}^{T_i} \max[c(t) - c, 0]dt \right\}, \tag{3}$$

where M is the number of days in the period under consideration, while the independent variable t varies in the interval from sunrise t_i to sunset T_i for the day $i, i = 1, 2, \ldots, M$. We reiterate here that c = 40 ppb is used as a threshold in our consideration.

Obviously, it is important to be able to control the error which appears when the continuous representation (3) is replaced by the discrete formula (2). Indeed, the discrete formula can be considered as a first order numerical method (the mid-point rule, see Marchuk, 1985) for the computation of an approximation of the integral in (3) with $\Delta t = 1$ hour. A higher order numerical algorithm [the Simpsons Rule, see (2) again] has also be applied with the same value of Δt in connection with a few model runs. These additional calculations indicate that the error is relatively small, about 6%, assuming here that the discrepancy between the results obtained by these two formulae gives us an evaluation of the error. This is the reason for which the discrete formula (1) is used in this study. It should be repeated here that the discrete formula (1) is commonly used for calculating AOT40 values.

3 Description of the numerical procedure

The idea we explore in our study is to *reconstruct* the *natural* dependence of the ozone concentrations on time, using hourly measurement data for O_3 obtained by several observation stations over Europe. Obviously, the terms *reconstruct* and *natural* are conditional. More precisely, we use mesh functions of measured values of ozone concentration $\{c_i\}_{i=1}^{n}$ as an interpolation points for cubic splines, assuming that such an interpolant reconstructs in a natural way the *real* function of ozone concentrations in time. There are reasons for such a consideration since the cubic spline has some nice conservation properties (DeVore and Lorentz, 1993). Their simplicity and smoothness are also adequate to the physical nature of the ozone concentration dynamics.

In order to describe our procedure we need to introduce some notations regarding cubic splines. The cubic spline consist of cubic polynomials pieced together in the interval

$$\Delta :\ a = x_0 < x_1 < \ldots < x_n = b \tag{4}$$

in such a fashion that their values and those of their first two derivatives coincide at the interior *knots* $x_i,\ i = 1, \ldots, n-1$. Consider a finite sequence $Y := (y_0, y_1, \ldots, y_n)$ of $n+1$ real numbers. Further we denote by $S_\Delta(Y;\cdot)$ an interpolating spline function S_Δ with $S_\Delta(Y;x_i) = y_i$ for $i = 1, \ldots, n$. To be uniquely determined by the sequence of the Y of support ordinates, we consider the following additional requirements (side conditions):

$$S^{'}_\Delta(Y;a) = y'_0,\ S^{'}_\Delta(Y;b) = y'_n, \quad \text{for given numbers } y'_0,\ y'_n \tag{5}$$

To calculate analitically the integral from the interpolating spline function $S_\Delta(Y;\cdot)$ it is essential to find its explicit form in each interval, i.e., the polinomial coefficients. In this section, following exactly Stoer and Bulirsch (2002), we will describe the computational method for determining the coefficients of the cubic spline functions whith prescribed values at their knots and satisfying the side conditions.

In what follows $\Delta = \{x_i \mid i = 0, 1, ..., n\}$ will be a fixed partition of the interval $[a, b]$ by knots $a = x_0 < x_1 < ...x_n = b$ and $Y = (y_i)_{i=1,...,n}$ will be a sequence of $n+1$ real numbers. In addition let I_j be the subinterval $I_j := [x_{j-1}, x_j]$, $j = 0, 1, ..., n-1$ and $h_{j+1} := x_{j+1} - x_j$ its length.

We refer to the values of the second derivatives at knots $x_j \in \Delta$,

$$M_j := S^{''}_\Delta(Y;x_j), \qquad j = 0, 1, \ldots, n, \tag{6}$$

of the desired spline function $S_\Delta(Y;\cdot)$ as the *moments* M_j of $S_\Delta(Y;\cdot)$. Now we will show that the spline functions are readily characterised by their moments.

The second derivative $S_\Delta(Y;\cdot)$ of the spline function coincides with a linear function in each interval $[x_j, x_{j+1}]$, $j = 0, 1, \ldots, n-1$, and that these linear functions can be described in terms of the moments M_j of $S_\Delta(Y;\cdot)$:

$$S^{''}_\Delta(Y;x) = M_j \frac{x_{j+1} - x}{h_{j+1}} + M_{j+1} \frac{x - x_j}{h_{j+1}} \quad \text{for } x \in [x_j, x_{j+1}] \tag{7}$$

By integration,

$$S^{'}_\Delta(Y;x) = -M_j \frac{(x_{j+1} - x)^2}{2h_{j+1}} + M_{j+1} \frac{(x - x_j)^2}{2h_{j+1}} + A_j, \tag{8}$$

$$S_\Delta(Y;x) = M_j \frac{(x_{j+1} - x)^3}{6h_{j+1}} + M_{j+1} \frac{(x - x_j)^3}{6h_{j+1}} + A_j(x - x_j) + B_j \tag{9}$$

for $x \in [x_j, x_{j+1}],\ j = 0, 1, \ldots, n-1$, where $A_j,\ B_j$ are constants of integration. From $S_\Delta(Y; x_j) = y_j,\ \ S_\Delta(Y; x_{j+1}) = y_{j+1}$ we obtain the following equations for these constants A_j and B_j:

$$M_j \frac{h_{j+1}^2}{6} \qquad + B_j = y_j \tag{10}$$

$$M_{j+1} \frac{h_{j+1}^2}{6} + A_j h_{j+1} + B_j = y_{j+1} \tag{11}$$

Consequently,

$$B_j = y_j - M_j \frac{h_{j+1}^2}{6} \tag{12}$$

$$A_j = \frac{y_{j+1} - y_j}{h_{j+1}} - \frac{h_{j+1}}{6}(M_{j+1} - M_j) \tag{13}$$

This yields the following representation of the spline function in terms of its moments:

$$S_\Delta(Y; x) = \alpha_j + \beta_j(x - x_j) + \gamma_j(x - x_j)^2 + \delta_j(x - x_j)^3 \tag{14}$$
$$\text{for } x \in [x_j, x_{j+1}]$$

$$\alpha_j := y_j, \qquad \gamma_j := \frac{M_j}{2}, \tag{15}$$

$$\beta_j := S_\Delta^{'}(Y; x_j) = -\frac{M_j h_{j+1}}{2} + A_j = \frac{y_{j+1} - y_j}{h_{j+1}} - \frac{2M_j + M_{j+1}}{6} h_{j+1} \tag{16}$$

$$\delta_j := \frac{S_\Delta^{'''}(Y; x_j^+)}{6} = \frac{M_{j+1} - M_j}{6h_{j+1}} \tag{17}$$

Thus $S_\Delta(Y; \cdot)$ has been characterised by its moments M_j and the last four formulas are implemented in our program for computation of the cubic spline coefficients in each interval $[x_j, x_{j+1}],\ j = 0, 1, \ldots, n-1$.

The calculation of the moments is adressed further in the last cited work. However, in our study we use for this task already prepared subroutines from Press et al. (1996) and that is why this part is skipped here.

Now, we can formulate the numerical procedure used in our study. The procedure contains several steps:

1. *Read* 15 values of ozone concentrations (from 07:00 CET to 21:00 CET) for each of the 183 days of the integration period from the file containing the hourly concentrations and from the valid measurements between 08:00 CET and 20:00 CET [as it is required by Directive 2002/3/EC, 2002]. In such a way we form the vectors xa (setting in time) and ya (values of the ozone concentrations).

2. *Compute* the first derivative at 08:00 CET using the values at 07:00 CET and 09:00 CET (The same computations we performed for the derivative at 20:00 CET). As stated above, we need them for the side conditions. If a measurement value is missing, we put the value of the derivative to zero.

3 *Compute* all the 13 values of the second derivative of the tabulated function and form a vector. We need it to compute the coefficients of the cubic spline.

4 *Compute* the coefficients of the spline $S(x)$ according to Stoer and Bulirsch (2002) and *form* a matrix $A(12, 4)$ containing the values for 12 intervals and four coefficients for each spline.

5 *Solve* the cubic equation $S(x) - 40 = 0$ and *find* the values of the real roots.

6 *Check* the sign of the expression $S(x) - 40$ in each subinterval, i.e., between the real roots.

7 As stated later, the AOT40 value can be treated as the sum of the positive signed areas, i.e., in the subintervals, where the expression $S(x) - 40$ is positive (see previous step). These areas are analytically calculated (because the coefficients of $S(x)$ are already known) one-by-one applying the Newton-Leibniz formulae for the definite integral.

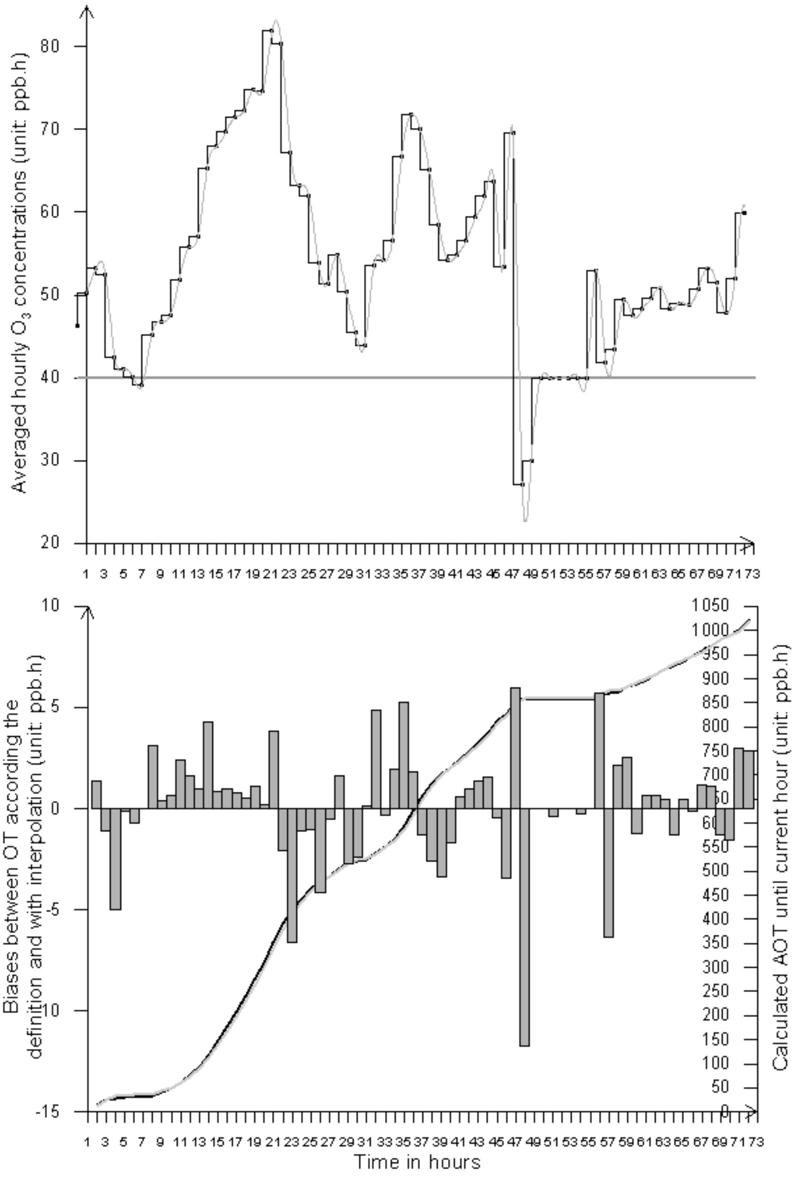
8 The procedure described above is performed for each of the 183 days and the results are written in order to be able to perform comparatative analysis with those achieved from the calculations according to the *classical* definition day-by-day.

4 Numerical results

In this section, we present the numerical results and describe how they have been obtained. To demonstrate the possibilities of the above-prescribed procedure, a dataset from 72 records of hourly ozone concentrations from the Ahtopol station on the Bulgarian Black Sea coast is used. Purposefully period with well expressed diurnal dynamics is selected, namely between 01 UTC on 24th of July to 00 UTC on 27th of July 2000 as shown on the upper pane of Figure 1. The period is longer then that prescribed in the directive, but this is performed only to demonstrate the differences of the two approaches. As many other quadrature formulas (1) and (3) have simple geometric interpretation.

The upper pane of Figure 1 shows the spline function through the knots – the hourly measurements. The geometric interpretation of formula (1) is the area between the 40 ppb – line and the step plot, and that of (3) – between this line and the smooth spline curve. Obviously these areas are not equal – in the periods of ascending concentration tendency (the morning hours, for example) the second is smaller and vice versa – in periods of descending trends (afternoon) the second is bigger. The difference becomes stronger in periods of rapid changes. On the lower pane of Figure 1 the biases between the two approaches are depicted with bars. The lines of the total AOT40 accumulation values, almost overlap in the two cases, which leads to the natural assumption, that these effects compensate each other due to the usual quasi-periodic diurnal dynamics. To examine this assumption however, it is necessary to extend the integration period, best up to the whole AOT40f definition period and to provide the calculations with data from stations with diverse ozone pollution patterns.

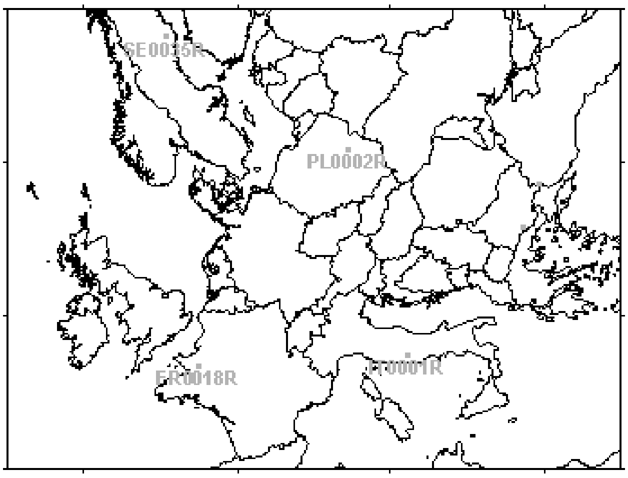
Figure 1 The geometric interpretation of the classical definition of the AOT40 (step plot) and proposed approach (smooth line) upper pane



Note: Biases (grey bars) between the two approaches and accumulated values according to the classical (in black) and proposed approach (in grey), lower pane.

We use EMEP data for our computations. EMEP stands for the European monitoring and evaluation programme. It is a scientifically-based and policy driven programme under the convention on long-range transboundary air pollution (CLRTAP) for international cooperation to solve transboundary air pollution problems. EMEP data are freely available for non-commercial use and through internet most of the observations can be accessed. To perform comparison between the results, obtained with the *classical* definition (1) and the continuous representation (3), datasets on hourly basis from four stations from this source have been used for the computations. To cover a greater circle of the concentration dynamics variety, the locations of the stations were selected in the different parts of Europe, as shown on Figure 2.

Figure 2 Map (EMEP domain fragment) of the location of the used measurement stations



Note: The EMEP stations are shown with their database signatures.

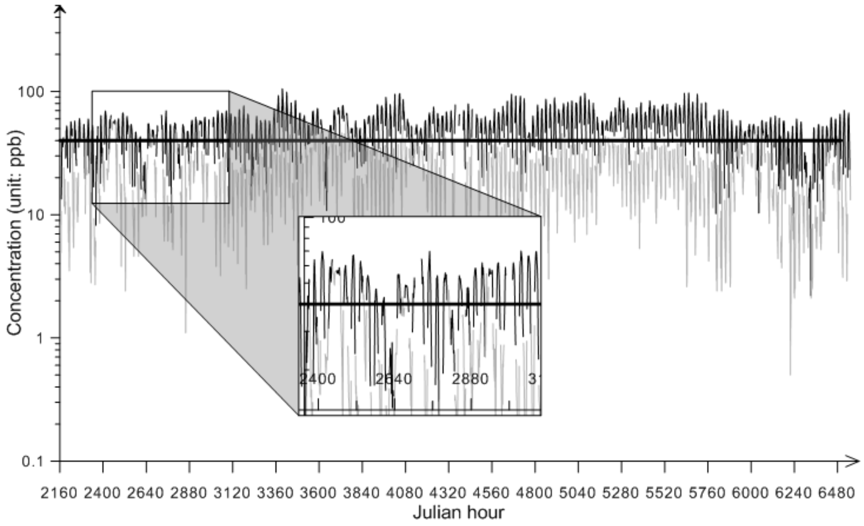
The time span of all datasets is the AOT40f definition period for the year 2009. Additionally, data from two Bulgarian stations Ahtopol and Rojen, for the year 2000, which are not included (for the year of 2000) in the EMEP-network, have been used. As mentioned above, station Ahtopol is on the Bulgarian Black Sea coast and station Rojen is situated in the montaneous southwestern part of the country, on place aproximately 1,600 m. above sea level. The time series of the two stations with the most drastic differences of the ozone dynamics are depicted on Figures 3 and 4. It can be observed that the ozone concentration in the Italian station is almost always above the threshold value of 40 ppb in all integration windows. This expresses the well known fact, that the ozone pollution is pronounced in regions with strong photochemical activity, such as the Mediterranean. The opposite case is for the Swedish station: relatively few measurements are above 40 ppb, the bigger part of all data are below this value.

Usually, there are missing data in all measurement data series. Very often they are grouped in 'gaps' of a few consecutive missing observations. A quantitative measure of this is the ratio of the number of valid data to all possible ones, called data completeness in this work. In such cases, where all possible measured data are not available, the following factor is used to calculate AOT40 values:

$$AOT40\,[estimate] = AOT40_{measured}\,\frac{total\,possible\,number\,of\,hours}{number\,of\,measured\,hourly\,values}$$

This procedure is prescribed in Directive 2002/3/EC (2002) and implicitly assumes linear accumulation of the AOT40 with time. Since the calculation routines of the continuous representation do not require equidistant location of the knots, the spline simply interpolates over the gaps. On the one hand, such an approach is much more consistent to the real concentration dynamics. On the other hand, it can lead to significant differences of the results, following the above, if the data completeness is relatively small and/or the missing data are grouped almost near the concentration maximum/minimum due to any systematic reason.

Figure 3 Time series of the ozone concentration in the period from the 1st April to 30th September 2000 for station Montelibretti (IT0001R)



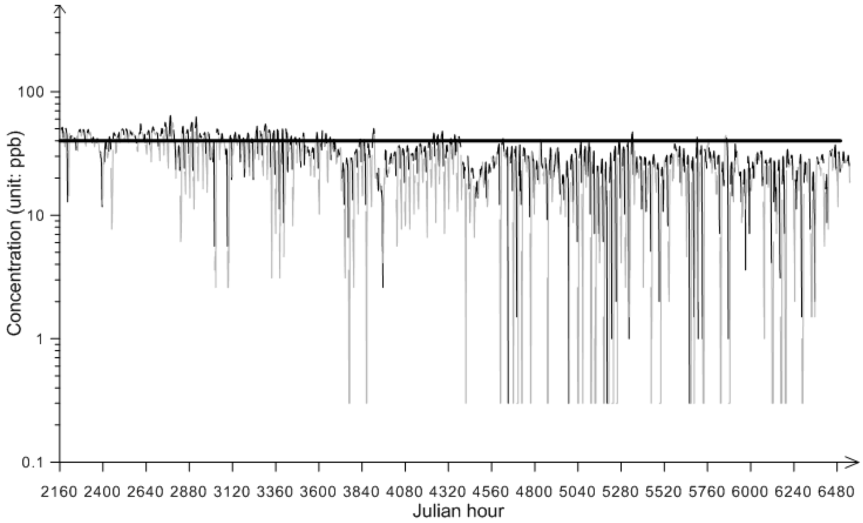
Note: The records in the AOT integration windows (08:00 to 20:00 CET) are shown in black, the remaining data in grey.

Some computational results are presented in Tables 1 and 2. The results of AOT40 values obtained for forest and crops by the classic, classical estimation, and by integration of the spline interpolation are tabulated. The computations are performed for measurement data obtained by several stations: Montelibretti, La-Coulonche, Vindeln, Jarczew, Rojen and Ahtopol. The data we have used was obtained for 2009 for the first four stations, and for 2000 for the last two stations.

5 Discussion and applicability

The analysis of the results presented in Section 3 shows that both approaches are applicable. At the same time the differences between the classical approach and the proposed one are higher for the periods of rapid changes of ozone concentrations. If the concentrations increase, then the *classical* definition gives higher values for AOT40. When the concentrations decrease, the proposed approach gives higher values for AOT40. For relatively short time periods (up to 48 hours) the relative difference of values computed by the *classical* definition and by the proposed approach may reach 15%.

Figure 4 Time series of the ozone concentration in the period from the 1st April to 30th September 2000 for station Vindeln (SE0035R)



Note: The records in the AOT integration windows (08:00 to 20:00 CET) are shown in black, the remaining data in grey.

Tables 1 and 2 show that the calculated values of the AOT vary up to one order for the AOTf and more than one for the AOTc between the most affected station Montelibretti and the station Vindeln, where the AOT-indexes are the smallest ones in comparison with the others. This circumstance is clear indication for the diverse pollution patterns in these places. Despite this, the AOT values, both for crops and forest integration period, calculated according the classical defnition, differ under 5% from their continuous parities for all stations.

Table 1 AOT40 values for forest obtained by classic, classical estimate definition and by spline interpolation

Station	*Year*	*AOTforests, ppb.h*			
		Data compl., %	*Classic*	*Class. est.*	*Spline*
Montelibretti	2009	98.7	29,531.4	29,920.3	29,224.2
LaCoulonche	2009	98.6	6,018.0	6,103.4	5,886.1
Vindeln	2009	99.8	3,065.0	3,071.1	3,013.9
Jarczew	2009	98.6	9,214.5	9,345.3	9,479.6
Rojen	2000	100.0	25,810.6	25,810.6	25,805.6
Ahtopol	2000	100.0	17,999.2	17,999.2	17,150.4

Table 2 Same as Table 1, but for AOT crops

Station	*Year*	*AOTcrops, ppb.h*			
		Data compl., %	*Classic*	*Class. est.*	*Spline*
Montelibretti	2009	98.3	17,558.5	17,862.1	17,405.5
LaCoulonche	2009	97.6	3,315.5	3,397.0	3,299.3
Vindeln	2009	99.7	1,342.5	1,346.5	1,333.0
Jarczew	2009	98.9	3,570.5	3,610.2	3,572.9
Rojen	2000	100.0	12,849.3	12,849.3	12,893.8
Ahtopol	2000	100.0	9,423.7	9,423.7	8,949.9

For relatively long periods the difference between the two approaches is getting smaller. Obviously, the reason for that is the quasi-cyclic nature of the dependance of ozone concentrations in time. Because of this quasi-cyclic nature, the real error of computing the AOT40 values is smaller than the value obtained using *a priori* quadrature error estimates.

The quasi-cyclic nature of the variation of ozone concentration in time is the reason that for long enough time periods the differences between values of AOT40 obtained by both approaches are very close. For the whole period of integration the maximum of the relative error is about 5% and it is reached for AOT40 crop values (measurement station of Ahtopol).

6 Conclusions

We have presented an alternative approach for computing exposure indices, such as AOT40, of the ground-level ozone. We have compared the alternative approach with the traditional way of calculating the exposure indices in order to discover possible weaknesses or strengths of both approaches. Cubic spline interpolation over data of ozone concentrations is considered as a tool for computing accumulated effects AOT(c). The definition of AOT(c) is considered as a quadrature formula for a real-life function describing the ozone concentrations in time. *A priori* and *a posteriori* error estimates are analysed. The obtained results by the proposed procedure are compared with those from the calculations by the classical definition. The computational results show that

1. the differences between the classical approach and the proposed one are higher for the periods of rapid changes of ozone concentrations; when the concentrations increase the classical definition gives higher values for AOT40
2. for relatively short time periods (up to 48 hours) the relative difference of values computed by the classical definition and by the proposed approach may reach 15%
3. for relatively long periods the difference between two approaches is getting smaller; the reason for that is the cyclic nature of the dependance of ozone concentrations on time.

The results obtained in this work by calculation of the AOT values using the classical definition do not relevantly differ from those obtained by the described in this work continuous representation, at least from the decision makers' point of view. This fact can be treated as indication for the computational applicability and consistency of the already widely used classical approach for quantitative assessment of the cumulative efect of the ozone pollution. Such procedure, as pointed above, can be considered as first order numerical method. It has some feedbacks and has to be applied carefully, especially on relatively short datasets or such with many missing data. On the other hand, however, routines based on diverse interpolation techniques, similar to the one proposed in this study, can also be recommended. One should take into account that the computations based on interpolation techniques could have more complicated implementations.

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