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Problem 5. Analytical solution for consolidation of a soil layer with finite thickness under cyclic mechanical loading

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1. Introduction. **GEO**|RUHR is a start-up in the field of geotechnical engineering. Among others we offer technology and scientific consultancy for the design of foundations of engineering structures. One of our business segments is the experimental and numerical subsoil analysis. In this framework we experimentally determine and evaluate soil parameters, which are necessary for the assessment of the subsoil behaviour as well as for the numerical modelling of foundation systems. Thereby, **GEO**|RUHR sets a focus on the analysis of engineering structures founded in soft soils under cyclic loading – e.g. foundation systems for on- and offshore wind turbines – as these systems are gaining increasing attention within the geotechnical community.

2. Objective. When founded on soft cohesive soils pore water dissipation and time evolution of settlements is a key issue in the analysis of relevant foundation systems, as these soils due to their low permeability show a retarded settlement behaviour. In order to do settlement and/or time prognoses for cyclically loaded foundation constructions an exact knowledge of the evolution of pore water pressure dissipation is important. For static loading this problem has been solved for many decades (see Terzaghi, 1923). However, for cyclic load applications as they can be found in the framework of on- and offshore foundation design this problem is not solved completely. By an experimental testing series **GEO**|RUHR is already able to do prognoses for the pore water dissipation behaviour. However, a comparison of the experimental data to an analytical solution of the consolidation equations is needed to validate the experimental testing results. This is requested as besides numerical methods, analytical solutions are strongly requested to verify the FEM results.

3. Mathematical Problem. An analytical solution for the consolidation process under cyclic loading exists in literature (see e.g. Barends, 2006, 2011). However, this approach assumes non-realistic boundary conditions as a soil stratum with infinite depth or a layer of high thickness are treated.

Therefore, a solution for a finite soil stratum, or shallow depth (see Fig 1) characterised by the following parameters is to be derived:

H	thickness of the stratum
k	hydraulic permeability of the soil
K_s	bulk modulus
α	compressibility of the solid phase
β	compressibility of the fluid phase
n	porosity
γ	volumetric gravity of the fluid phase

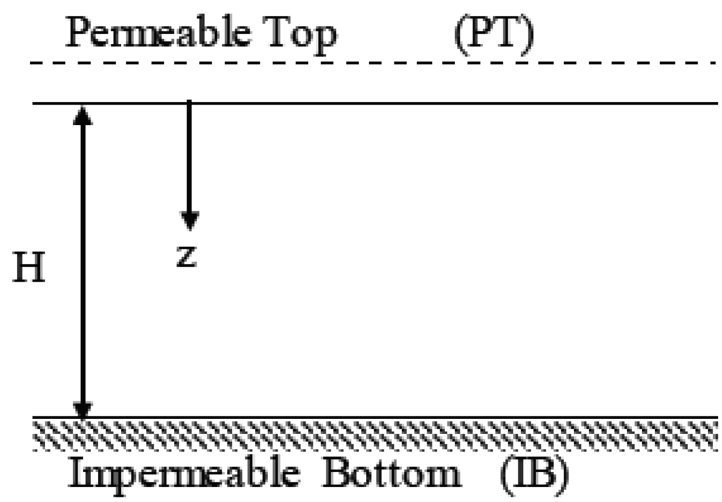

Figure 1. Soil stratum with given boundary conditions (permeable top, impermeable bottom – PTIB)

A vertical, cyclic load is applied to the top of the stratum, e.g. as haversine function of time or other:

$$L(t) = q\sin^2\left(\frac{\pi t}{d}\right) \quad \text{or} \quad L(t) = q\cos\left(\frac{\pi t}{d}\right),$$

where

$L(t)$	loading function
t	time
q	load amplitude
d	load period

The one-dimensional consolidation equation is given by

$$\frac{\partial u}{\partial t} = C_z \cdot \frac{\partial^2 u}{\partial z^2} + \frac{\alpha}{\alpha + n\beta} \cdot \frac{dL\,(t)}{dt}.$$

This equation describes the behaviour of the excess pore water pressure u with time and along the depth z. The consolidation coefficient C_z is given as:

$$C_z = \frac{k \cdot K_s}{\gamma(1 + \frac{n\beta}{K_s})}.$$

The boundary conditions may be given as follows

$$u\,(0, t) = 0, \quad \frac{\partial u}{\partial z}\,(H, t) = 0.$$

The initial condition is given as

$$u\,(z, 0) = 0.$$

The task is to find an analytical solution to the above formulated boundary value problem. Next, based on the analytical solution to evaluate the following sub-tasks:

1. excess pore water pressure as a function of time and depth $u(z,t)$;
2. explicitly derive the phase shift ψ between excess pore water pressure $u(z,t)$ and the applied load with time (for a fixed depth) especially at the bottom ψ $(z = H,\ t \to \infty)$, the phase shift or lag is a positive or negative delay of the excess pore water pressure as compared to the applied surface load that may vary with depth;
3. parameter analysis for the solution regarding permeability k as a function of relevant parameters (stratum height H, bulk modulus K_s, phase shift (see 2.), load amplitude q and loading period d).
4. parameter analysis for the phase shift ψ as a function of the fluid and solid phase compressibility and soil permeability.