The 2D/3D Best-Fit Problem

EngView Systems Sofia

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113th European Study Group with Industry

Problem Description

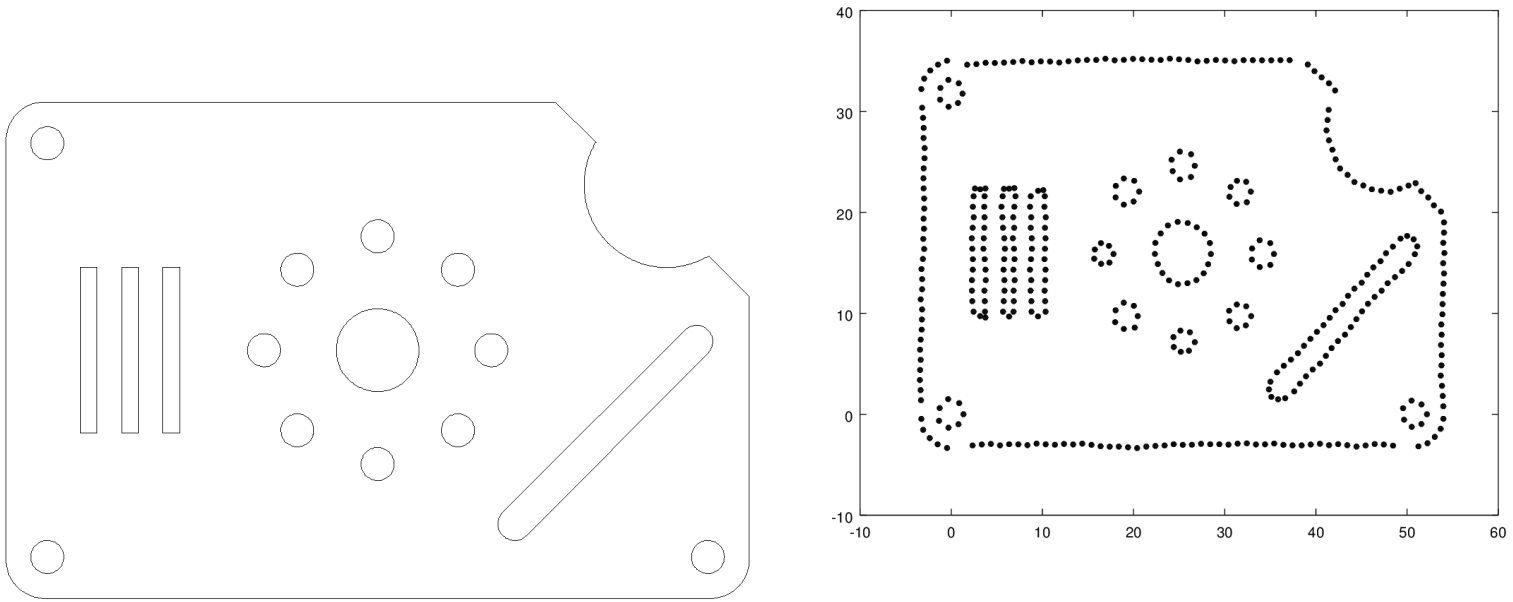
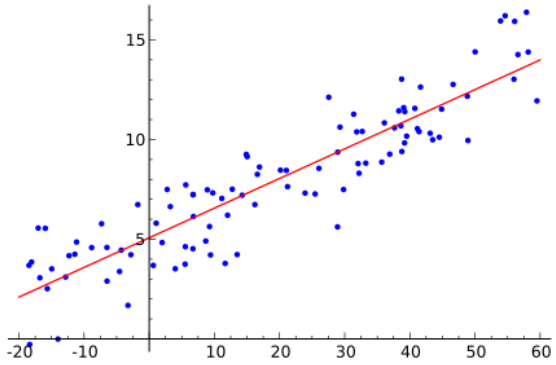


Figure: Left: CAD Model. Right: Point cloud obtained via EngView Systems' scanner.

Karhunen-Loeve Transform

$G(x_G, y_G)$ – mass center.
We look for a line
$A(x - x_G) + B(y - y_G) = 0$ that minimizes the sum of squared Euclidean distances



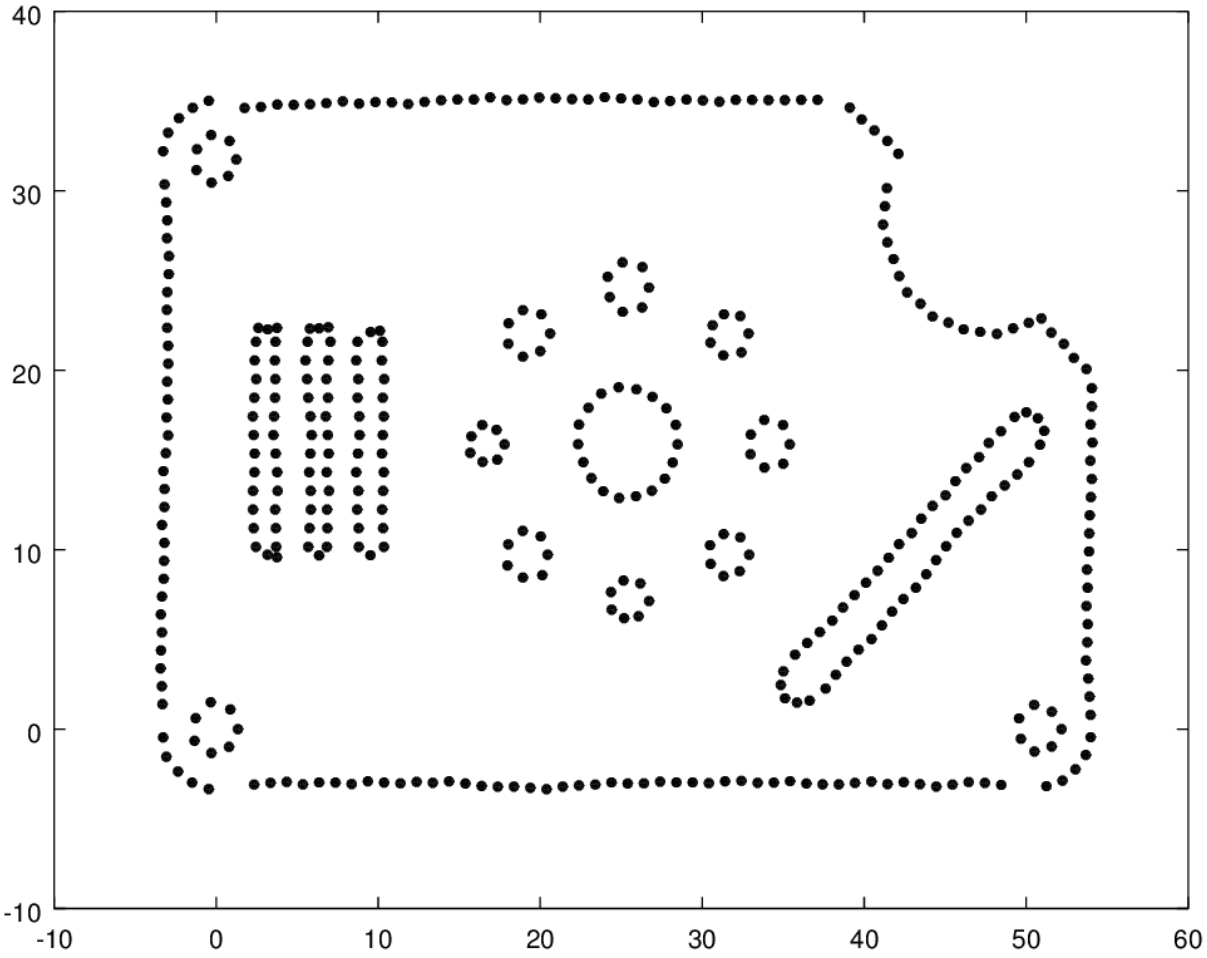
$$\sum_{i=1}^{N} \left(A(x_i - x_G) + B(y_i - y_G)\right)^2 \to min \qquad s.t. \quad A^2 + B^2 = 1.$$

Quadratic optimization on the unit circle:

$$\left\langle \begin{pmatrix} \langle x, x\rangle & \langle x, y\rangle \\ \langle y, x\rangle & \langle y, y\rangle \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}, \begin{pmatrix} A \\ B \end{pmatrix} \right\rangle \to min \qquad s.t. \quad A^2 + B^2 = 1$$

$$\Rightarrow \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = v_1, \text{ normalized eigenvector, corresponding to } \lambda_1.$$

Karhunen-Loeve Transform - Example



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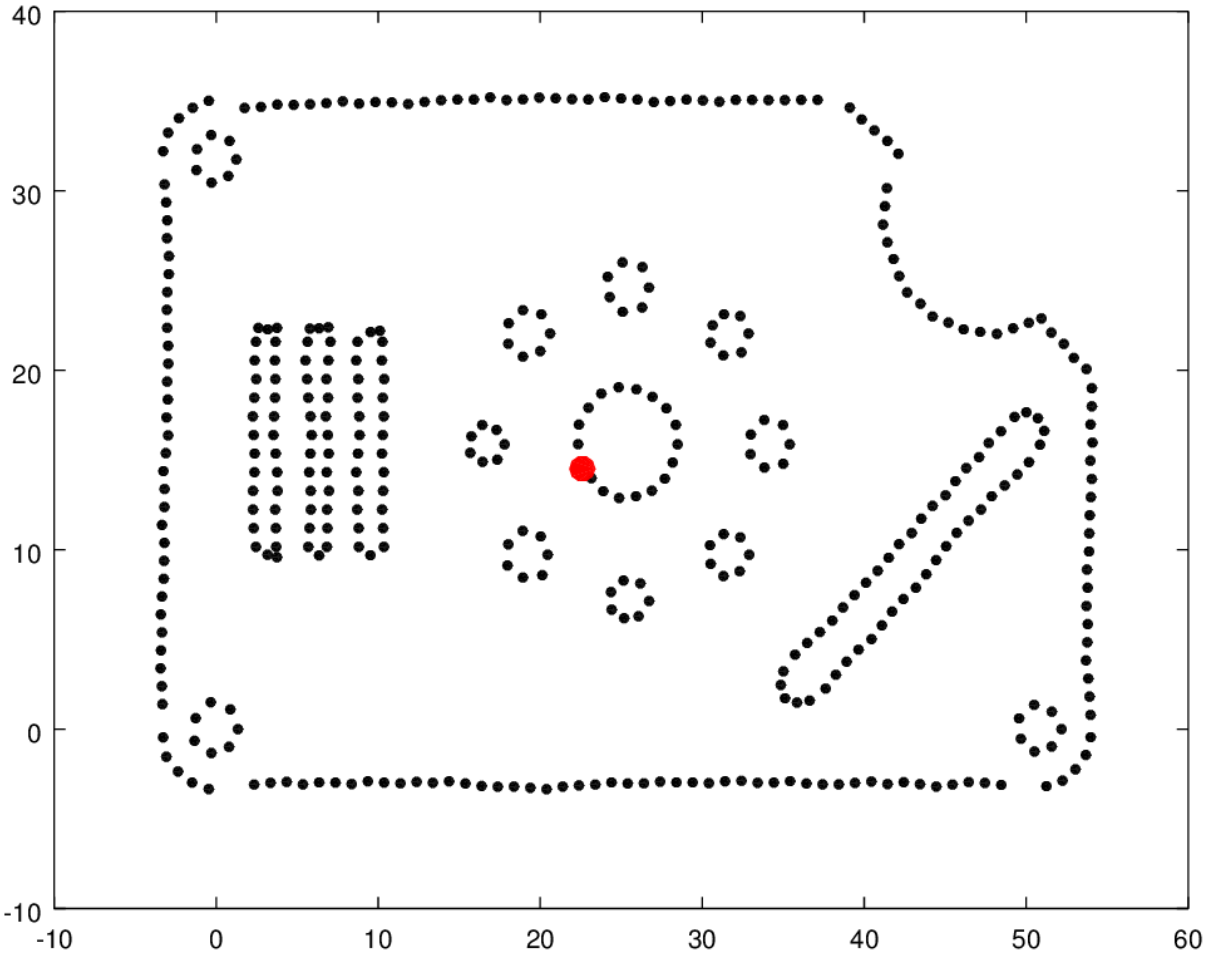

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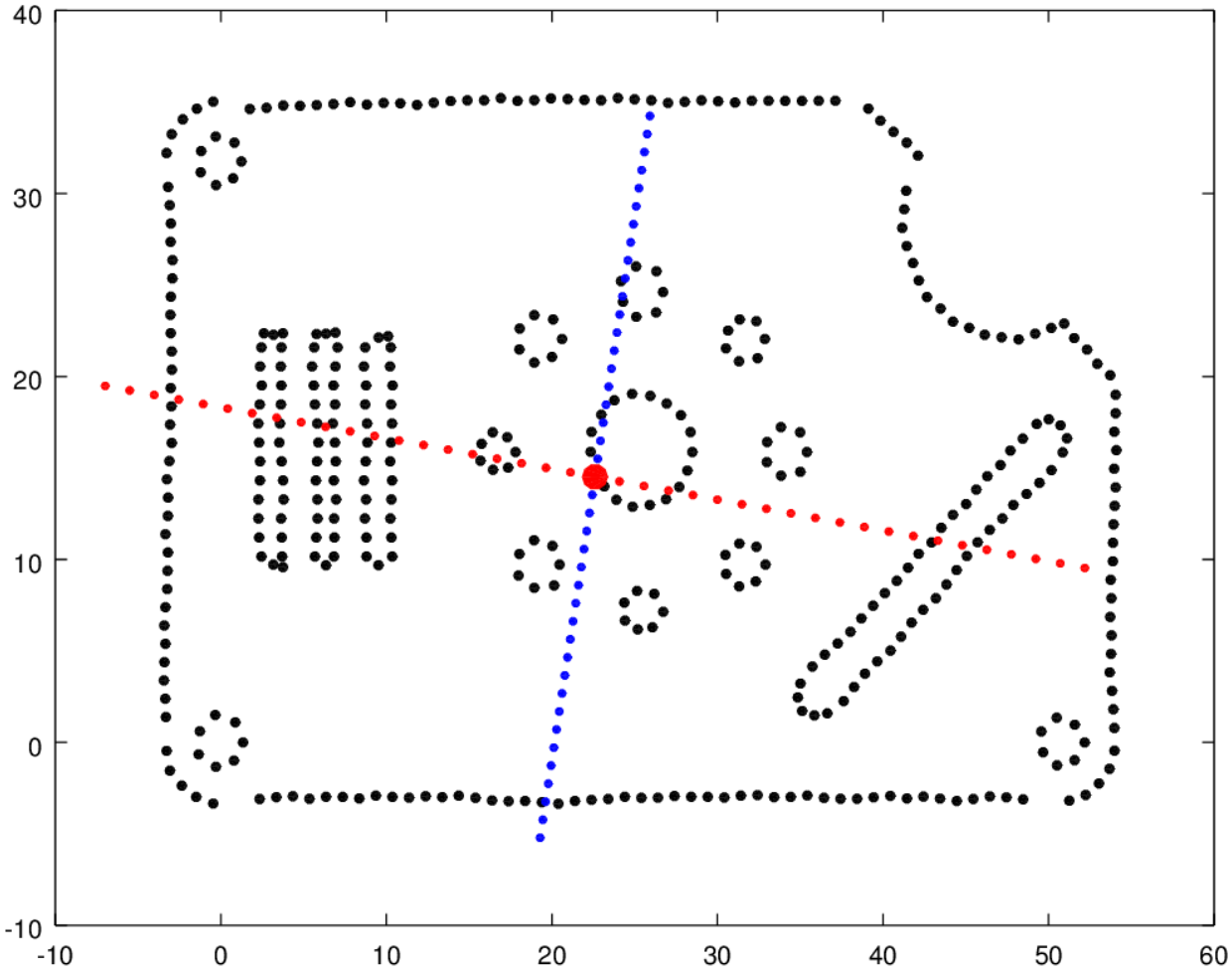
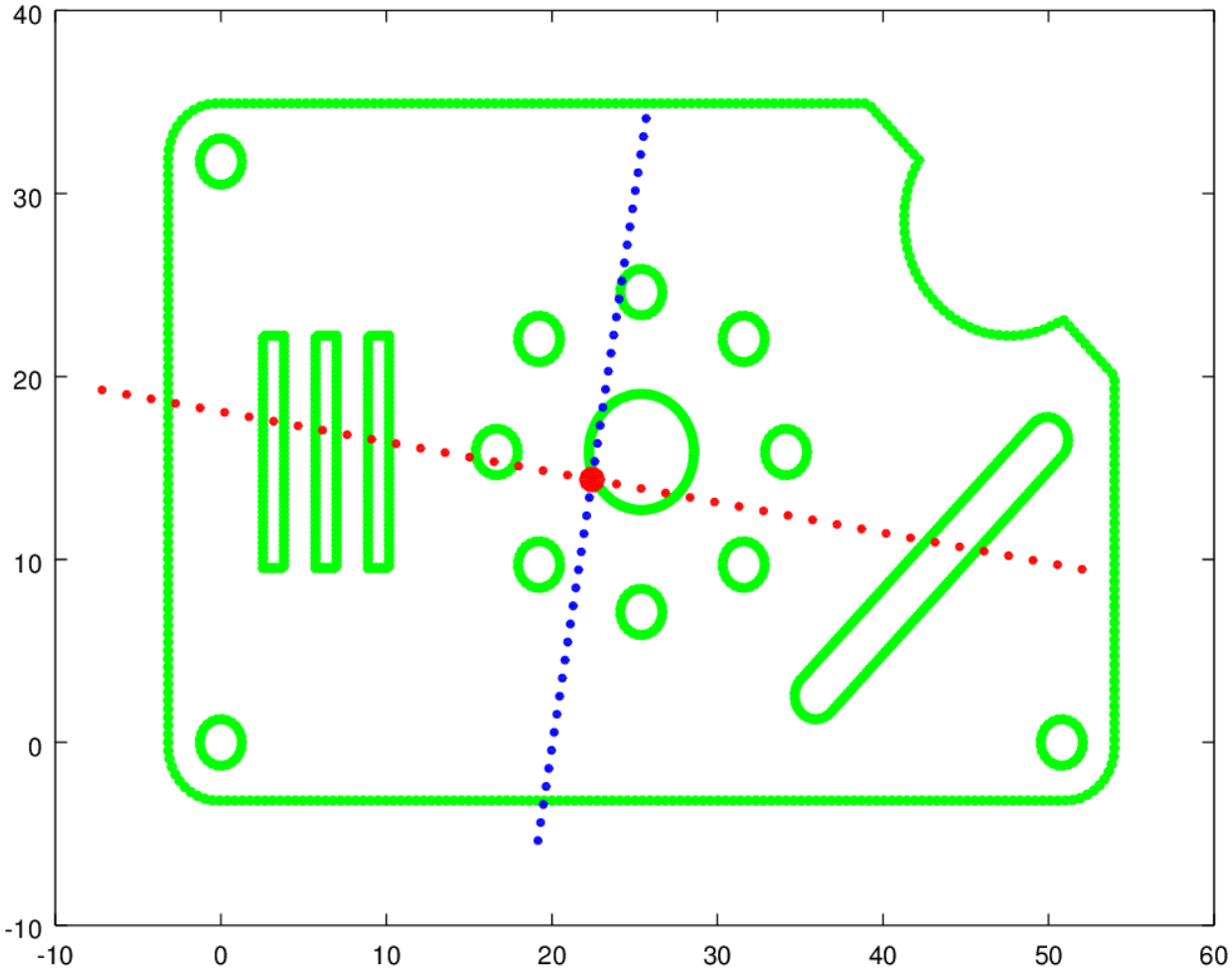


Data Fitting - Example 1: "Good" scanned data



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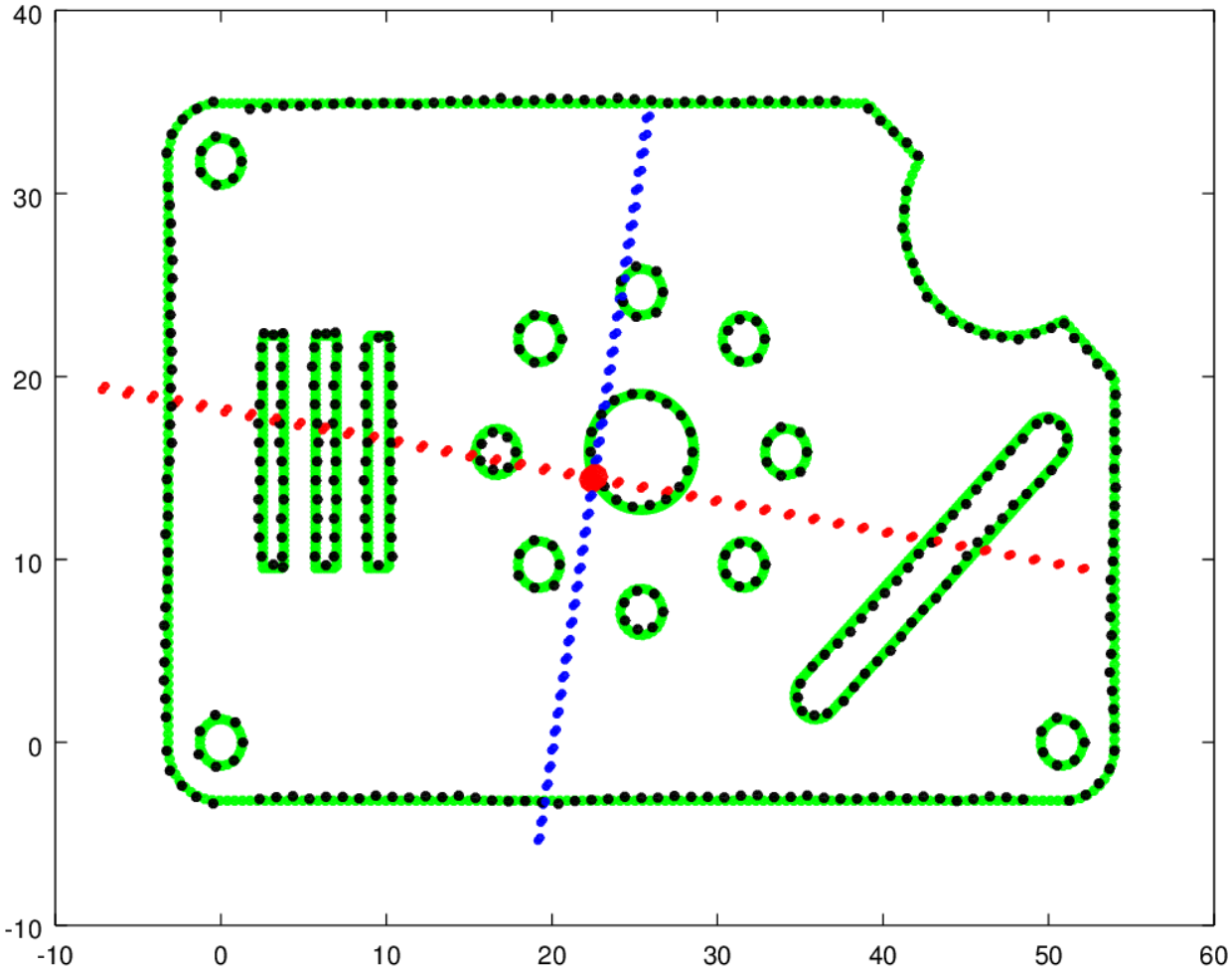
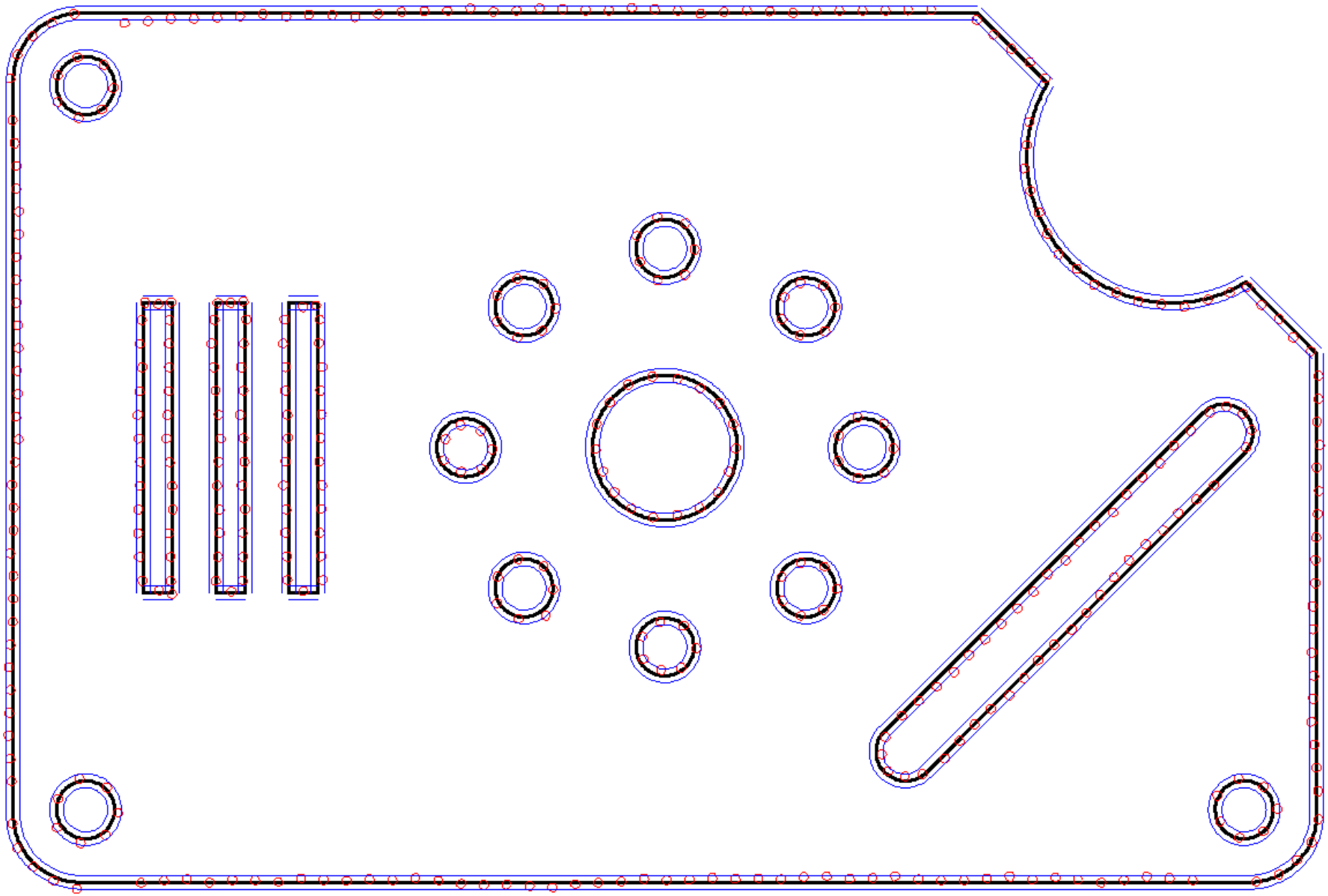


Axes Orientation Check



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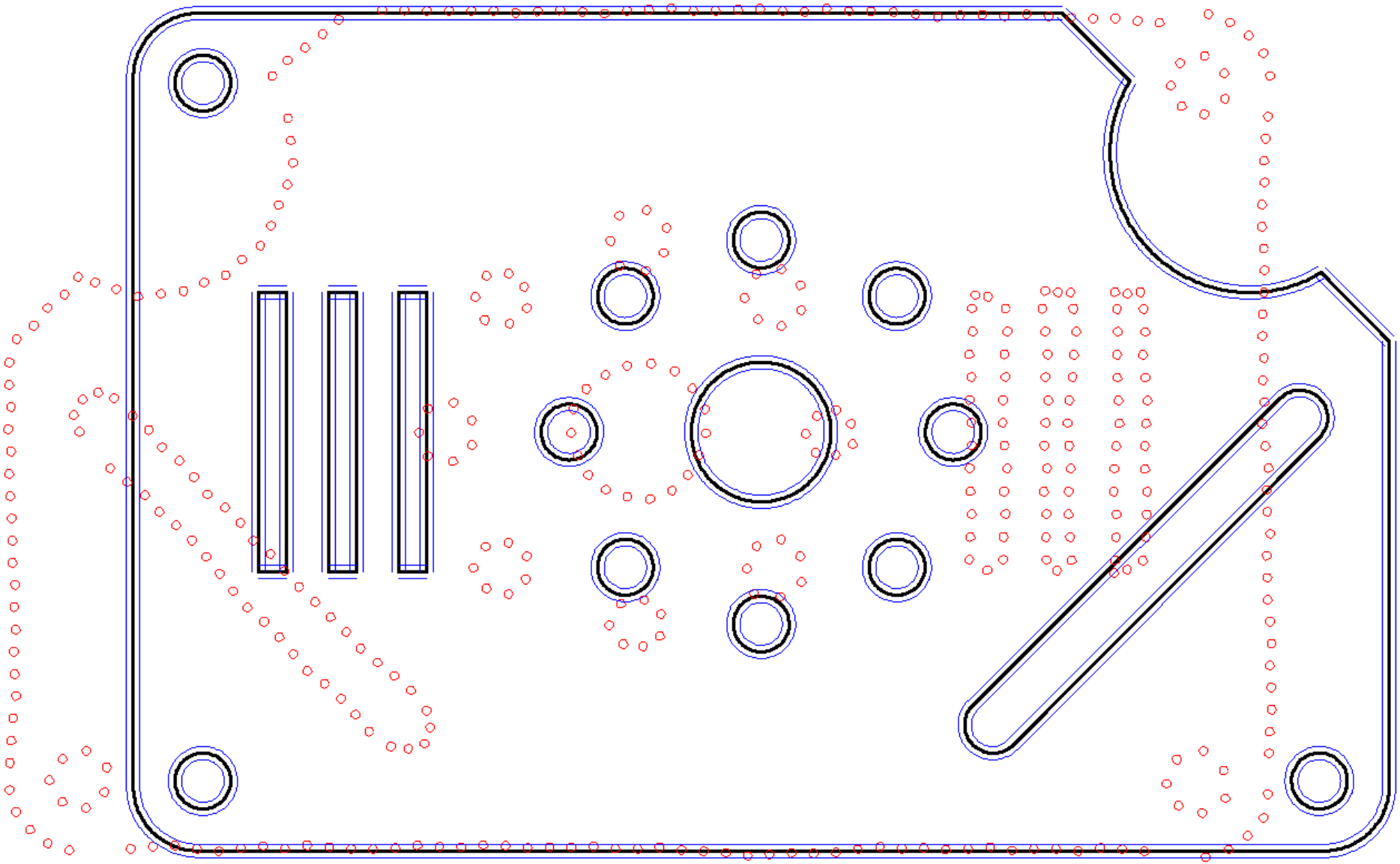

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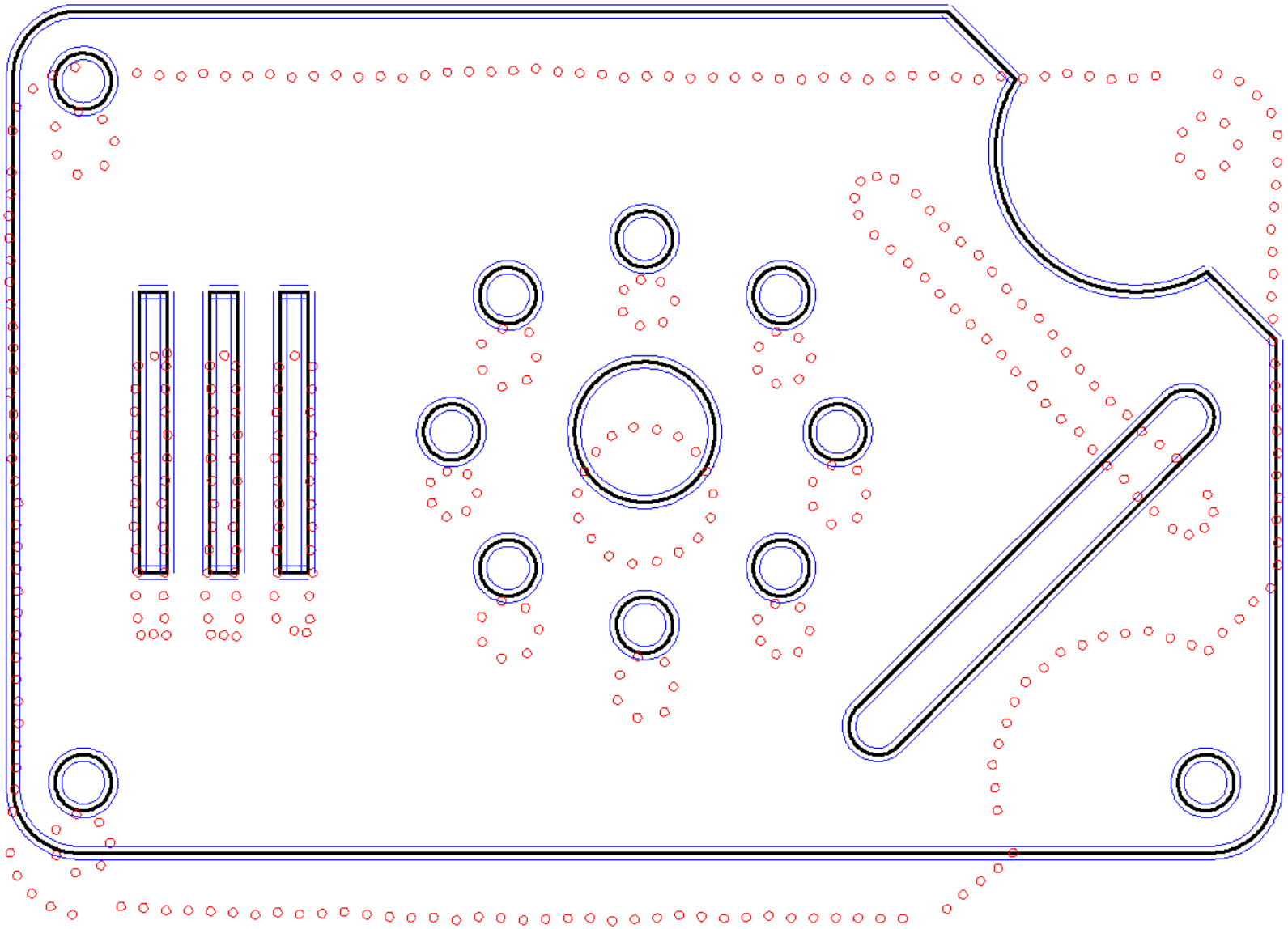

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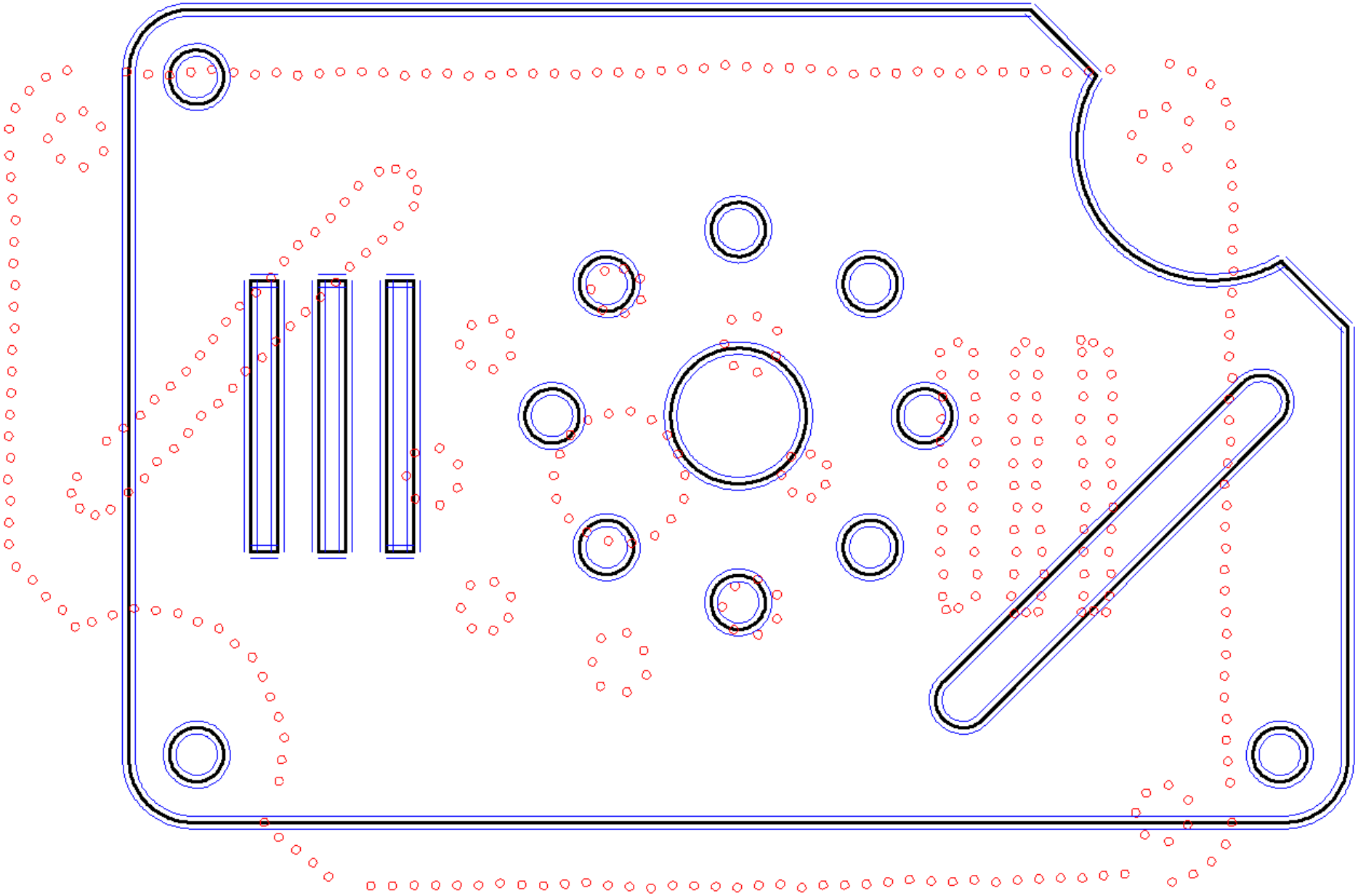

Algorithm for aligning the coordinate systems

Input: **cad_data**, **sc_data**
Output: **sc_data_aligned**

1. Compute the mass centers G^{CAD} and G^{sc} of **cad_data** and **sc_data**.
2. Compute the shift $\vec{d} = G^{CAD} - G^{sc}$.
3. Apply the Karhunen-Loeve Transform to both data sets.
4. Compute the rotation angle $\theta = \arccos\langle v_1^{CAD}, v_1^{sc} \rangle$.
5. Derive **sc_data_aligned** from **sc_data** via translation by $\vec{d}$ and rotation on θ around G^{CAD}
6. FUZZ the **cad_data**.
7. Axes orientation check.

Data Fitting - Example 2: "Bad" scanned data

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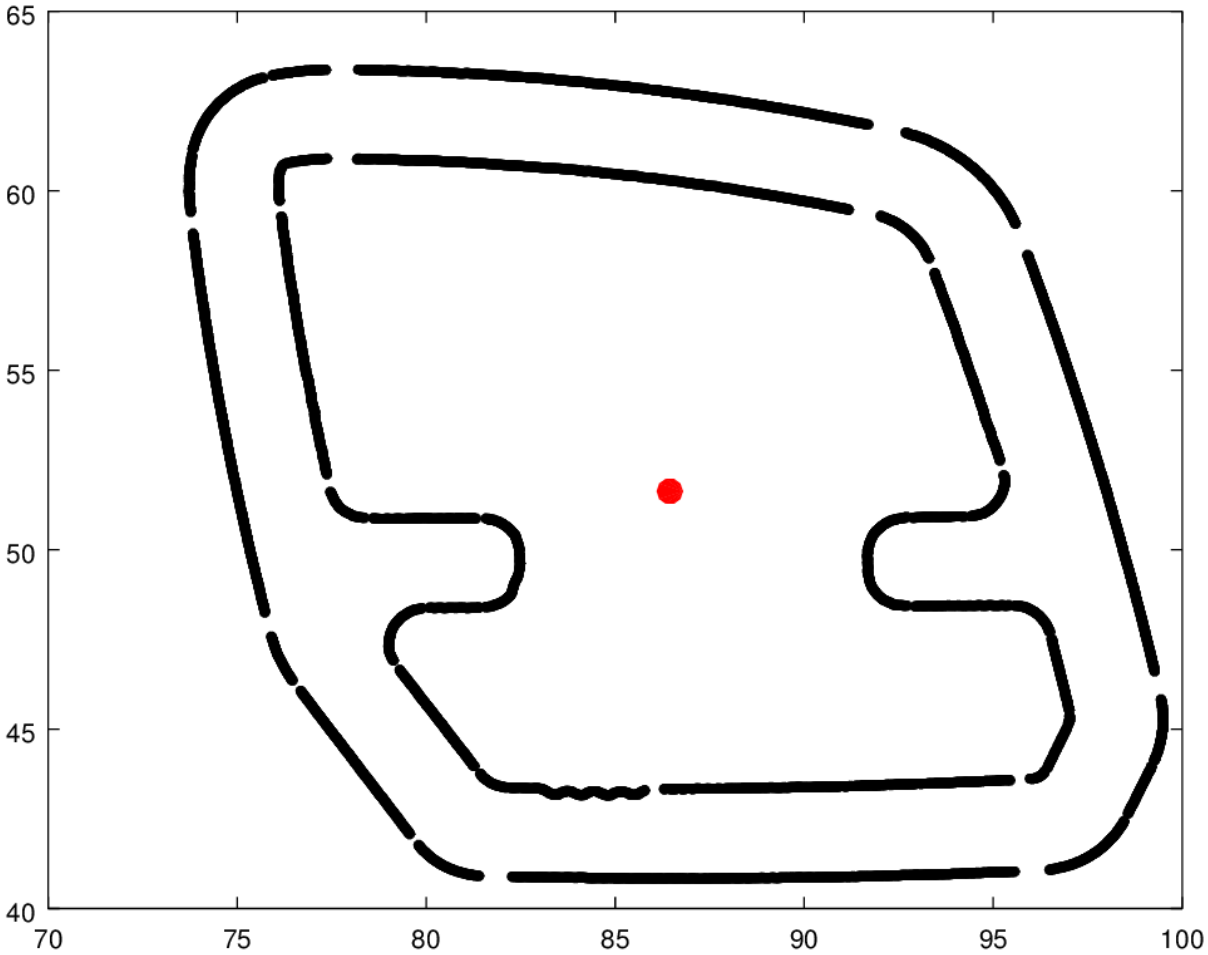

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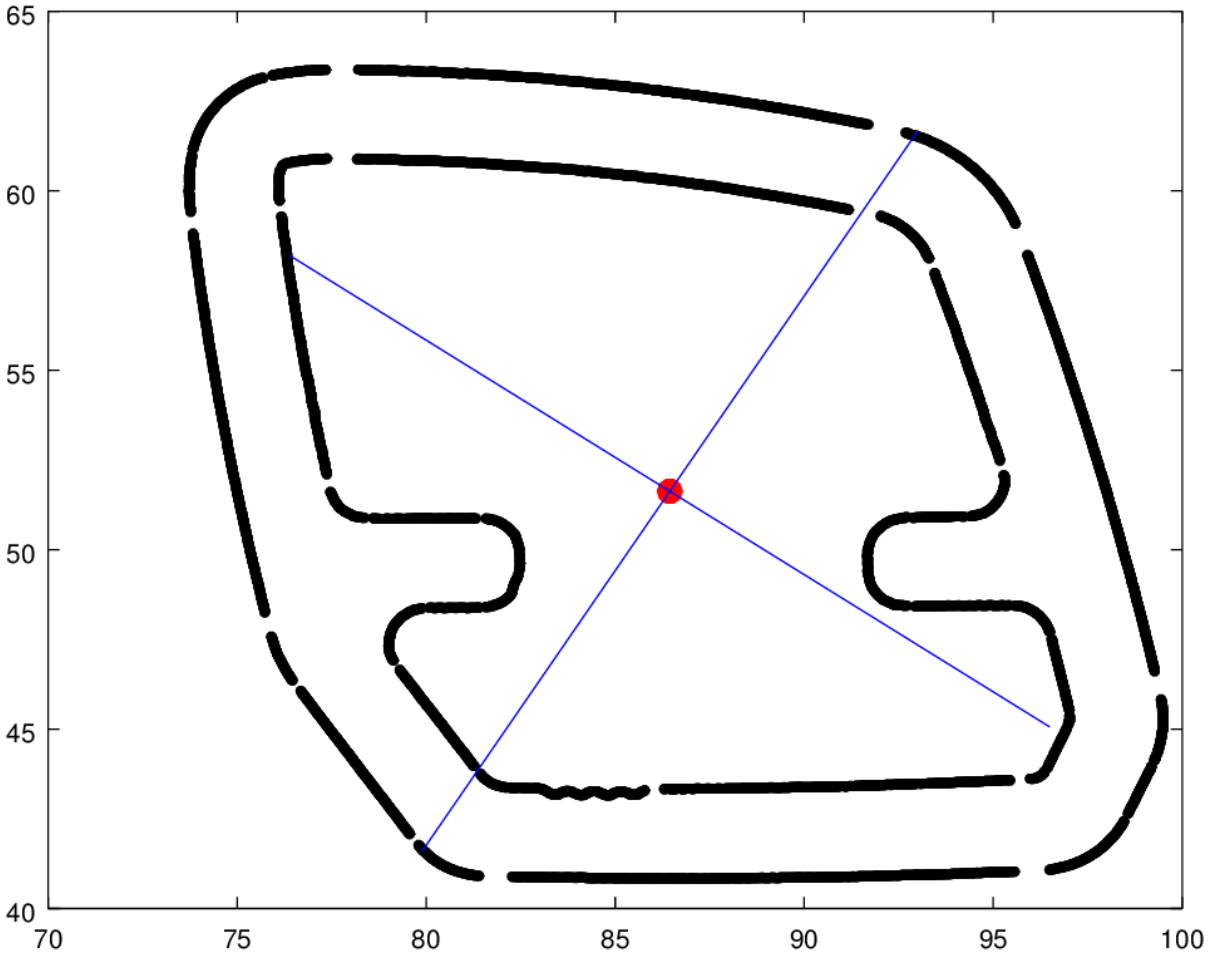

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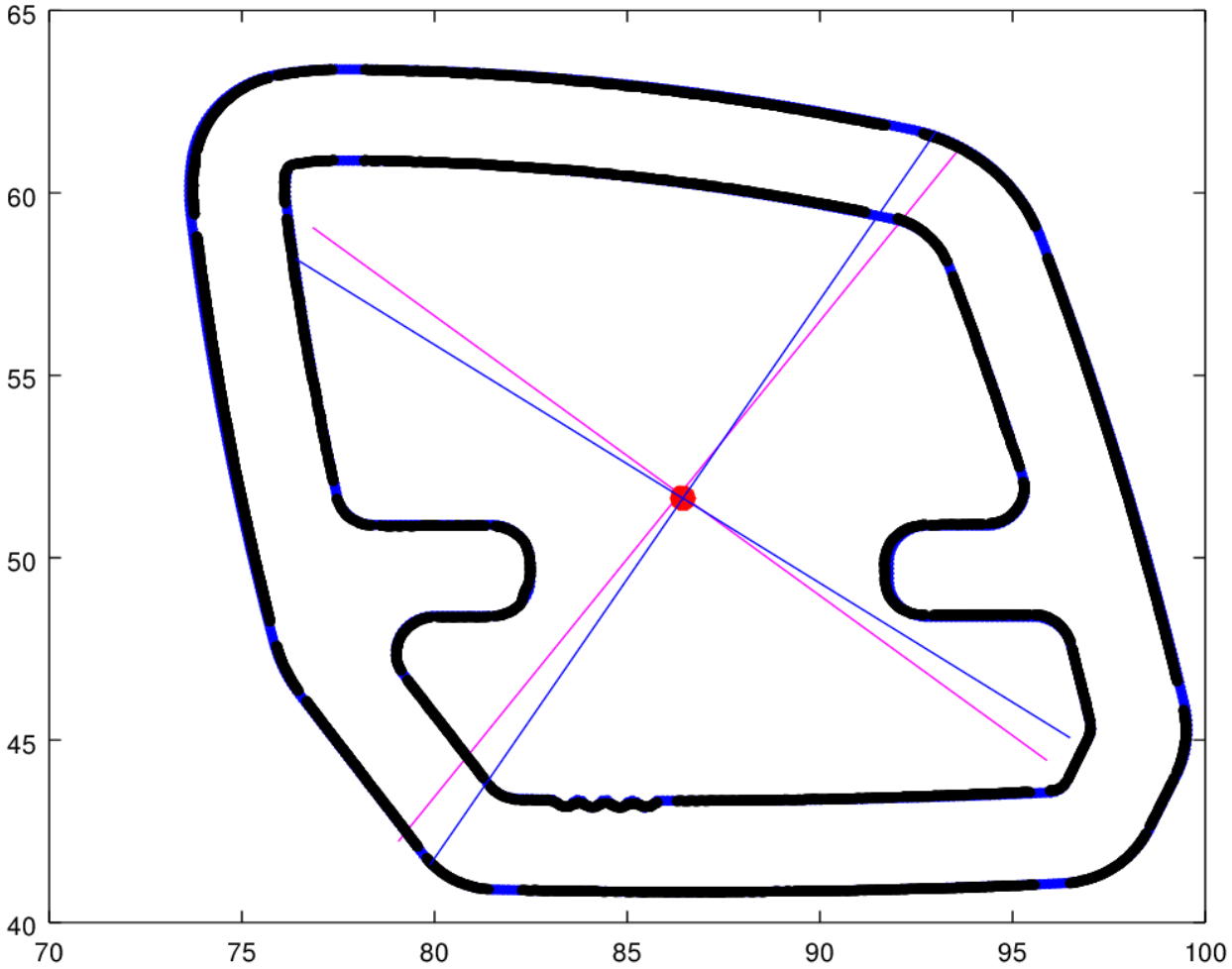

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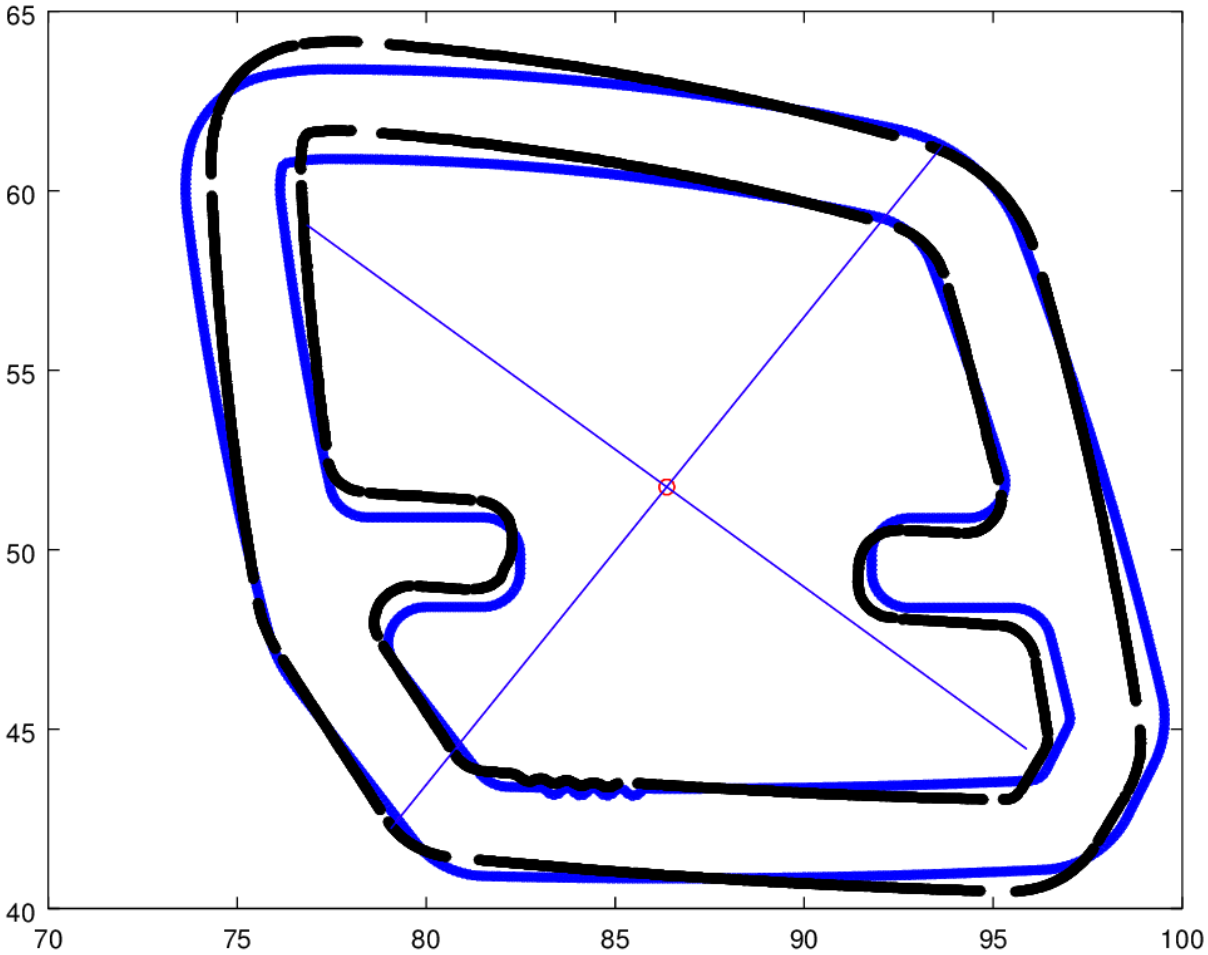

Future Work

- Additional local procedure in Algorithm 1 for improving the data aligning (e.g., angle bisection method).
- Point-to-point similarity measure for the aligned data sets.
- Comparing CAD data with a scanned part of it.
- The 3D case.

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Thank you for your attention!