# Spline Intersection Improvement

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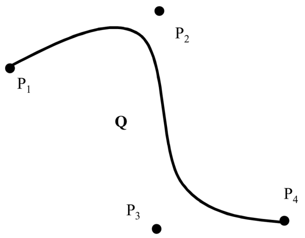
# Outline

- Description of the problem
- Proposed solution
- Obtained results
- Exceptions
- Analytical solution
- Investing if a ray intersects a hair

# Description of the Proposed Problem

- Each hair can be composed by many spline primitives. The light illuminates our hair.
- We have to model spline curve primitives with 4 control points in 3D space $p_0; p_1; p_2; p_3$.
- Each control point has its own width of the curve $w_0; w_1; w_2; w_3$.
- Spline curve center as function of curves evolution parameter is described with
  $\vec{p}(u) = p_3 u^3 + 3p_2 u^2(1-u) + 3p_1 u(1-u)^2 + p_0(1-u)^3$



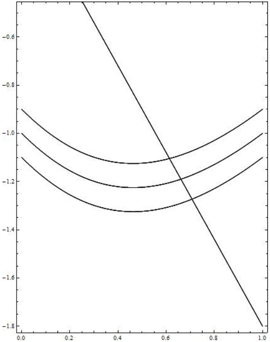
# Description of the Proposed Problem

$$\vec{p}(u) = p_3 u^3 + 3p_2 u^2(1-u) + 3p_1 u(1-u)^2 + p_0(1-u)^3 \quad , u \in [0,1]$$

$$\vec{p}(u) = \begin{pmatrix} p_x(u) \\ p_y(u) \\ p_z(u) \end{pmatrix}$$

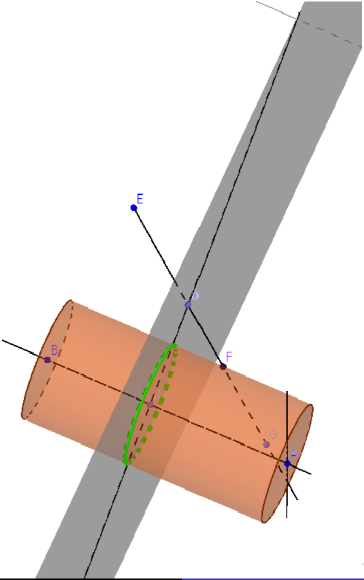
$$w(u) = w_3 u^3 + w_2 u^2(1-u) + w_1 u(1-u)^2 + w_0(1-u)^3$$

$$\vec{v}(t) = \vec{o} + t\vec{d}$$



# Proposed Solution

http://tube.geogebra.org/student/m1622695



# Proposed Solution

http://tube.geogebra.org/student/m1622695

$u_i = \frac{i}{N}, \quad i = 0, \ldots, N.$

For a fixed $u_i$ we find the corresponding $t_i$ from the equation

$$[\vec{r}(t_i) - \vec{p}(u_i)]\frac{d\vec{p}(u)}{du} = [\vec{o} + t_i\vec{d} - \vec{p}(u_i)]\frac{d\vec{p}(u)}{du} = 0$$

Or equivalently

$$[o_x + t_i d_x - p_x(u)]\frac{dp_x(u)}{du} + [o_y + t_i d_y - p_y(u)]\frac{dp_y(u)}{du} + [o_z + t_i d_z - p_z(u)]\frac{dp_z(u)}{du} = 0$$

# Proposed Solution

Having the point from the ray $\vec{r}(t_i)$, we calculate how far it is from the point $\vec{p}(u_i)$.
$|\vec{r}(t_i) - \vec{p}(u_i)|^2 = [t_i d_x - p_x(u_i)]^2 + [t_i d_y - p_y(u_i)]^2 + [t_i d_z - p_z(u_i)]^2$
When we have that

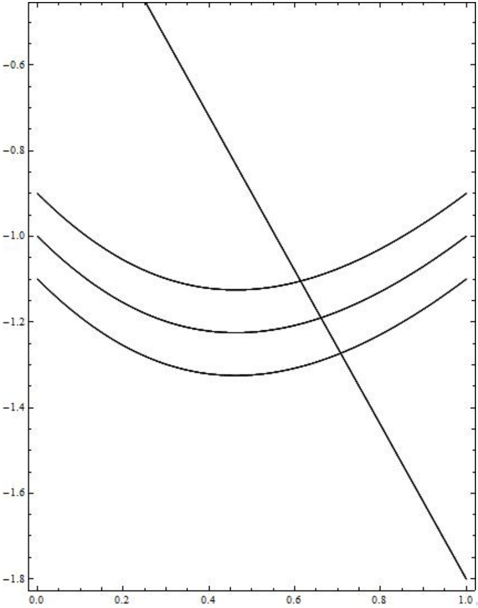
$$|\vec{r}(t_{i-1}) - \vec{p}(u_{i-1})|^2 > w(u_{i-1})^2$$

and

$$|\vec{r}(t_i) - \vec{p}(u_i)|^2 < w(u_i)^2$$

this means that somewhere between $u_{i-1}$ and $u_i$ (correspondingly $t_{i-1}$ and $t_i$) the ray intersects the hair. Then we can break the interval $[u_{i-1}, u_i]$ into subintervals with the points $v_0, \dots, v_{N_1}$ and repeat the algorithm for the interval $[v_0, v_{N_1}]$ instead of $[0, 1]$.
**Remark.** We have to find **all** such couples $(u_{i-1}, u_i)$.

# Obtained results



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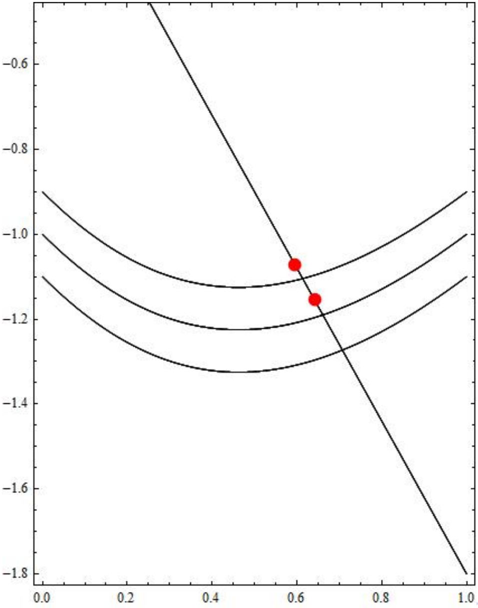

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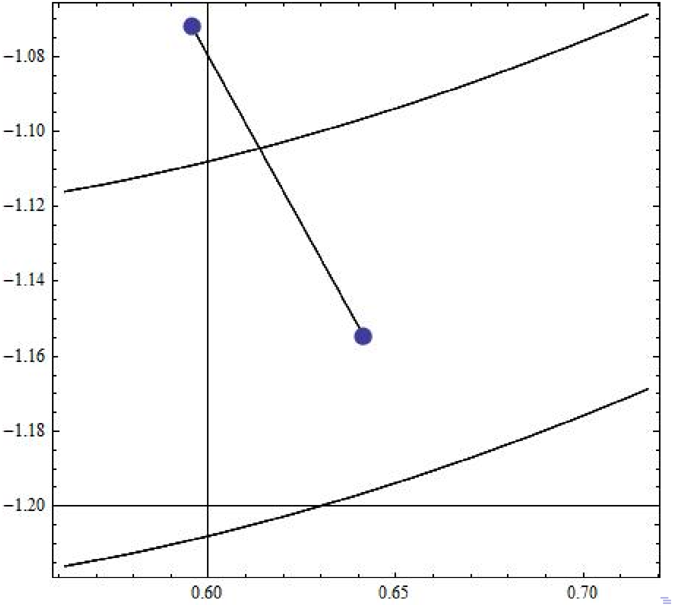

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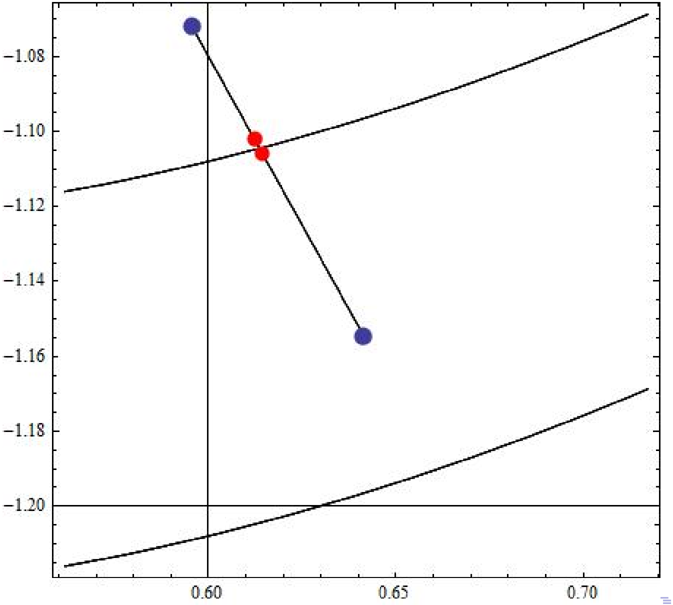
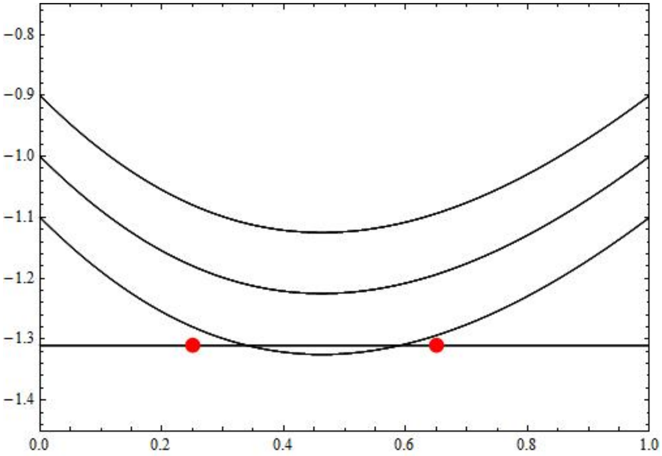


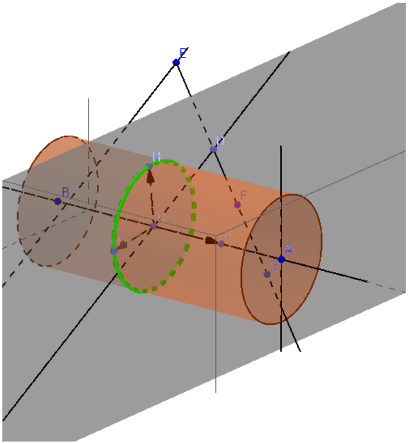
# Problems




Searching for suspicious points:

$$|\vec{r}(t_i) - \vec{p}(u_i)|^2 < w(u_i)^2 + \epsilon$$

http://tube.geogebra.org/material/simple/id/1634965



# Analytical solution

$$\vec{d} = (d_x, d_y, d_z),\ \vec{o} = (o_x, o_y, o_z),\ \vec{p}(u) = (p_x(u), p_y(u), p_z(u))$$

$$\frac{d\vec{p}(u)}{du} = \vec{q} = (q_x, q_y, q_z)$$

$$\vec{a} = (a_x, a_y, a_z) = (0, -q_z, q_y), \quad \vec{b} = \vec{q} x \vec{a} = (b_x, b_y, b_z)$$

The surface of the hair has the following equation

$$\vec{s}(u, v) = \vec{p}(u) + w(u).cos(v)\vec{a} + w(u).sin(v)\vec{b}, \ \ u \in [0, 1],\ v \in [0, 2\pi)$$

# Analytical solution

$$\vec{s}(u,v) = t\vec{d}$$

$$p_x + w.cos(v)a_x + w.sin(v)b_x = o_x + td_x$$
$$p_y + w.cos(v)a_y + w.sin(v)b_y = o_y + td_y$$
$$p_z + w.cos(v)a_z + w.sin(v)b_z = o_z + td_z$$

$$w.cos(v)a_x + w.sin(v)b_x = o_x + td_x - p_x$$
$$w.cos(v)a_y + w.sin(v)b_y = o_y + td_y - p_y$$
$$w.cos(v)a_z + w.sin(v)b_z = o_z + td_z - p_z$$

$$w^2.cos^2(v).(a_x^2 + a_y^2 + a_z^2) + w^2.sin^2(v).(b_x^2 + b_y^2 + b_z^2)$$
$$+ 2.cos(v).sin(v)(a_x b_x + a_y b_y + a_z b_z)$$
$$= t^2 - 2Bt - C_1$$

# Analytical solution

$$t^2 - 2Bt - C = 0$$

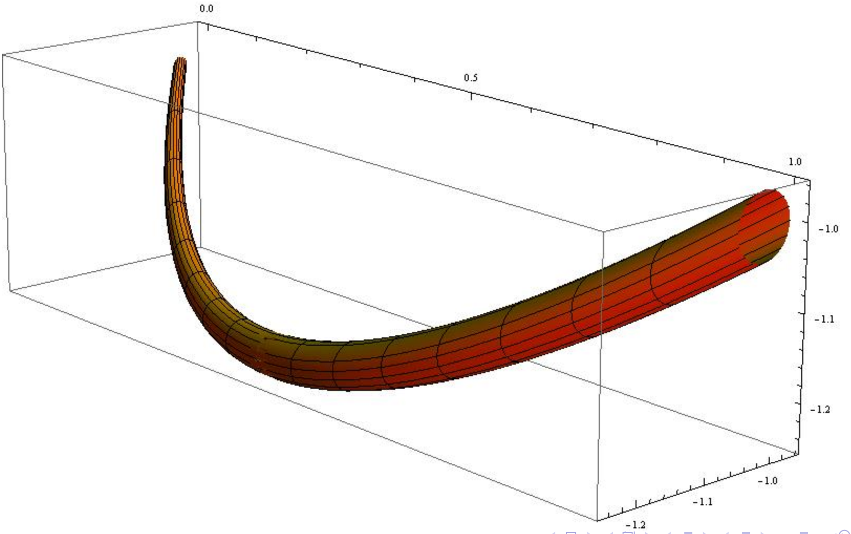
$$D = B^2 + C$$

If $D > 0$ we have two roots (and two intersections): $t_1 = B + \sqrt{D}$, $t_2 = B - \sqrt{D}$.
If $D = 0$ we have one root (one intersection): $t_0 = B$.
If $D < 0$ we have no roots.
Actually since $B = B(u)$ and $C = C(u)$ then the first line is an equation of the type

$$F(u,t) = 0 \ \ , (u,t) \in [0,1]x[T_0, T_n]$$

# Does the ray intersect the hair

$t\vec{d}$ belongs to the cuboid for $t \in [T_0, T_N]$.



# Does the ray intersect the hair

For simplicity of the calculations let $\vec{o} = (0, 0, 0)$ and $|\vec{d}|^2 = 1$. Then $\vec{r}(t) = t\vec{d}$. If the ray intersects the hair then there is a point $(u_0, t_0)$ (many points but one is enough) for which

$|t_0\vec{d} - \vec{p}(u_0)|^2 < w(u_0)^2$ Or equivalently

$$|t_0\vec{d} - \vec{p}(u_0)|^2 - w(u_0)^2 < 0$$

$$t^2 - 2t[dxp_x(u) + dyp_y(u) + dzp_z(u)] + \vec{p}(u)^2 - w^2(u) < 0$$

$$G(u, t) < 0$$

If the absolute minimum of $G(u, t)$ in the rectangle $[0, 1]x[T_0, T_N]$ is less than zero then the ray intersects the hair.

Thank you for your attention!!!